Homework II Solutions

1. There are several varieties of retro-reflectors that are commercially available; one type is comprised of transparent spheres, the backs of which are silvered. Light is refracted at the front surface, focused onto the rear surface, and there reflected back out in the direction it came. Determine the necessary index of refraction of the sphere. Assume the incident light is collimated.

\[
\frac{n_{a\rightarrow v}}{s_0} + \frac{n_s}{s_i} = \frac{n_s - n_{a\rightarrow v}}{R}
\]

\[
\frac{1}{\infty} + \frac{n_s}{2R} = \frac{n_s - 1}{R}
\]

\[\rightarrow n_s = 2\]
2. Regarding your own face as a real object, describe the image of your face which you see 3 feet from the center of (and looking directly toward) a polished and well-reflecting brass ball 1 foot in diameter hanging in front of a pawn shop. Determine the image graphically and by calculation.

![Diagram of light rays and reflection](image)

**Graphically:** 3 rays: (1), (2), (3) ....

**Calculation:**

\[
\frac{1}{S_o} + \frac{1}{S_i} = -\frac{1}{f}
\]

\[
\therefore \quad S_i = -0.227 \text{ ft}
\]

\[
M = \frac{-S_i}{S_o} = 0.091
\]

The image is virtual \((S_i < 0)\), upright \((M > 0)\), and reduced \(|M| < 1\) at 0.227 ft from vertex towards face.
3. Suppose you have a concave spherical mirror with a focal length of 10 cm. At what distance must an object be placed if its image is to be erect and one and a half times as large? What is the radius of curvature of the mirror?

\[ f = 10 \text{ cm} \quad \text{and} \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \]

\[ s_i = -\frac{3}{2} s_o \]

\[ \frac{1}{s_o} = \frac{2}{3} \frac{1}{s_o} = \frac{1}{10} \]

\[ s_o = \frac{10}{3} = 3.33 \text{ cm} \]

\[ s_i = -\frac{3}{2} s_o = -\frac{3}{2} \cdot \frac{10}{3} \text{ cm} = -5 \text{ cm} \]

\[ 2f = \frac{2}{s} = 2 \times 10 = 20 \text{ cm} \]
4. What is the smallest refracting angle that a glass prism ($n = 1.5$) can have so that no ray can be transmitted through it? What is the angle for a water prism ($n = 1.33$)?

\[
\frac{n}{\sin \left( \frac{\delta}{2} \right)} = \frac{\sin \left( \frac{\delta + \delta}{2} \right)}{\sin \left( \frac{\delta}{2} \right)}
\]

\[
n \sin \frac{\delta}{2} = \sin \left( \frac{\delta}{2} \right) \cos \left( \frac{\delta}{2} \right) + \cos \left( \frac{\delta}{2} \right) \sin \left( \frac{\delta}{2} \right)
\]

Divide eqn. by \( \cos \left( \frac{\delta}{2} \right) \)

\[
tan \left( \frac{\delta}{2} \right) = \frac{\sin \left( \frac{\delta}{2} \right)}{n - \cos \left( \frac{\delta}{2} \right)}
\]

\[
L = 2 \tan^{-1} \left[ \frac{\sin \left( \frac{\delta}{2} \right)}{n - \cos \left( \frac{\delta}{2} \right)} \right]
\]

if no ray transmitted then \( \delta > 90^\circ \)

Min. angle for total internal reflection: \( \delta = 90^\circ \)

With \( n = 1.5 \), \( \delta = 90^\circ \):

\[
L = 2 \tan^{-1} \left[ \frac{\sin \left( 90^\circ \right)}{1.5 - \cos \left( 90^\circ \right)} \right] = 83.27' 13''
\]

\[
L (\text{water prism}) = L (n = 1.33) + 97^\circ 14' 46''
\]
5. Find the image of the object for a single concave mirror system shown on the back page by (a) measuring the radius $R$ and calculating the focal length for the concave mirror, (b) drawing the focal point into the diagram, and (c) constructing the image of the object graphically with three rays.

$R = 10 \text{ cm, and } f = \frac{R}{2} = 5 \text{ cm}$

<table>
<thead>
<tr>
<th></th>
<th>Incident Rays</th>
<th>Reflected Rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Parallel to the optical axis</td>
<td>Through the focal point of the mirror (F)</td>
</tr>
<tr>
<td>2</td>
<td>Through the center of the mirror (C)</td>
<td>Through the center of the mirror (C)</td>
</tr>
<tr>
<td>3</td>
<td>Through the focal point of the mirror (F)</td>
<td>Parallel to the optical axis</td>
</tr>
</tbody>
</table>

Three rays: