Homework Solutions IV

1. (A) From the given location of C1 and C2 and the values of R1, R2, n, and d of the thick lens shown in Fig. 1, determine its focal length, the location of its focal points, and its principal planes. (B) Draw your results into the diagram and obtain graphically the location and size of the image. What is $s_0$ and $s_i$ and the magnification $M$ in numerical value and sign?

(A) $n = 2.0$, $d = 3.0 \text{ cm}$, $R_1 = -10.0 \text{ cm}$ (Left of $V_1$), $R_2 = -4.0 \text{ cm}$ (Left of $V_2$)

The thick lens equations in the page 6 of Handout I:

$$\frac{1}{f} = (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1) d}{n R_1 R_2} \right] \Rightarrow f = 5.3 \text{ cm}$$

$$h_1 = -\frac{f(n-1)d}{R_2 n} = +2.0 \text{ cm} \text{ (Rightside from } V_1)$$

$$h_2 = -\frac{f(n-1)d}{R_1 n} = +0.8 \text{ cm} \text{ (Rightside from } V_2)$$

$$s_0 = +3.5 \text{ cm} \Rightarrow s_i = -10.2 \text{ cm} \text{ (Virtual image at 10.2 cm from } H_2)$$

$$M = -\frac{s_0}{s_i} = +2.9 \text{ (Up-right and about 3 times magnified image)}$$

From the ray-tracing,

$$s_i = 10.2 \text{ cm on the left side from } H_2 \text{ (Virtual image)}$$

$$y_o = 1.1 \text{ cm and } y_i = 3.3 \text{ cm} \Rightarrow M = 3.0$$

********** Good agreement with calculation.
2. Solve ray tracing problems (A), (B), and (C) in Fig. 2, treating all lenses as thin lenses.
(C)
Find the point where the two incoming beams intersect behind lens.
3. The eye of a particular person is described (strongly over-simplified) by a positive lens with a focal length which can be varied between 1.45 and 1.55 cm, and a distance of this lens to the retina of 1.6 cm.

(A) What is the range (far point and near point) of good sight for this eye?
(B) What spectacles (focal lengths? refractive power?) will correct the eyesight as well as possible to the normal one?—and how does this eye-glass change far point and near point vision of the person?
(C) What would be the visual magnification if the person uses a magnifying glass of F = 5 cm with or without spectacles?

Besides giving the answers to questions (A) and (B), try to make some schematic sketches for illustration.

(A) What is the range (far point and near point) of good sight for this eye?

Lens equation for a positive lens:

\[
\frac{1}{f_{\text{eye}}} = \frac{1}{s_0} + \frac{1}{s_i}
\]

For this eye, \(s_i = 1.6\) cm and \(f_{\text{eye}}\) can vary from 1.45 to 1.55 cm. Therefore, \(s_0\) can vary from 15.5 cm to 49.6 cm to make clear image on the retina.

**Near point = 15.5 cm, Far point = 49.6 cm**
(B) What spectacles (focal length? refractive power?) will correct the eyesight as well as possible to the "normal" one — and how does this eye-glass change far point and near point vision of the person?

* Normal eye usually means that its far point is at infinity.

Since the far point of this eye is 49.6 cm (not infinity), the eye is **nearsighted**. Therefore, a negative lens \((f = f_{\text{cor}})\) is needed to correct this eye. When \(s_o = \infty\), the lens combination of \(f_{\text{eye}} = 1.55\) cm and the corrective lens \(f_{\text{cor}}\) should make \(s_i = 1.6\) cm, that is, clear image on the retina. (We ignore the distance between the eye lens and the corrective lens.)

\[
\frac{1}{f_{\text{eye}}} + \frac{1}{f_{\text{cor}}} = \frac{1}{s_o} + \frac{1}{s_i} \Rightarrow f_{\text{cor}} = -49.6 \text{ cm}
\]

**Refractive power** \(= f_{\text{cor}}^{-1} = -2.0\) Diopters \((= -2.0 \text{ m}^{-1})\)

Then, the **near point** \((f_{\text{eye}} = 1.45\) cm) changes to **22.5 cm**.

(C) What would be the visual magnification if the person uses a magnifying glass of \(f = 5\) cm with or without spectacles?

**Magnifying Glass:** \(M = \frac{d_o}{f}\) where \(f = 5\) cm (Or if you use \(M = \frac{d_o}{f} + 1\), \(M\) is larger by 1.)

Without spectacles: \(d_o = \text{Near point} = 15.5\) cm \(\Rightarrow M = 3.1\) (Or, \(M = 4.1\))

With spectacles: \(d_o = 22.5\) cm \(\Rightarrow M = 4.5\) (Or, \(M = 5.5\))
4. The ocular (or eyepiece) of an optical instrument is composed of two identical converging lenses of focal length 5 cm each, separated by 2.5 cm. Find the position of the foci of the system as measured from the closer lens.

For \( L_1 \):
\[
\frac{1}{s_{11}} + \frac{1}{s_{01}} = \frac{1}{f} \quad \Rightarrow \quad s_{11} = s_{01}f(s_{01} - f)
\]

For \( L_2 \):
\[
\frac{1}{s_{02}} + \frac{1}{s_{22}} = \frac{1}{f} \quad \Rightarrow \quad s_{22} = s_{02}f(s_{02} - f)
\]

And
\[
s_{02} = d - s_{11} \quad \Rightarrow \quad s_{22} = (d - s_{11})f(d - s_{11} - f)
\]

Substitute the first equation into the last to find \( s_{22} \):
\[
s_{22} = (d - s_{11})f(d - s_{11} - f) = \left[ f \cdot d \cdot (s_{01} - f) - f \cdot s_{01} \right] / [(d - f) \cdot (s_{01} - f) - f \cdot s_{01}]
\]

\( s_{22} \rightarrow \infty \) gives position of front focus, as measured from the first lens:
\[
f_f = f(d - f) / (d - 2f) = 5/3 \text{ cm}
\]

\( s_{01} \rightarrow \infty \) gives position of back focus, as measured from lens \( L_2 \):
\[
f_b = f(d - f) / (d - 2f) = 5/3 \text{ cm}, \text{ or } 2.5 + 5/3 = 4.17 \text{ cm from lens } L_1.