Huyghens - Fresnel principle

describes propagation of wavefronts through material.

- any undisturbed point on wavefront serves as a source of spherical secondary wavelets (of same frequency as primary wave). Amplitude of resulting optical field at any point beyond is the superposition of all secondary wavelets (considering their amplitudes and relative phases). Amplitude of secondary wave depends on angle $\theta$ with respect to propagation direction.

$$E = E_0/2 \left(1 + \cos \theta \right)$$; no wave propagating backwards!

- Primary wavefront at a later time is envelope of all secondary wavelets. Wavelets advance with speed and frequency equal to that of primary wave at each point in space.
Illustration of H.-T. principle with refraction process:

Wavellet from point B travels to D in time $\Delta t$ with velocity $v_i$. During the same time, wavellet energy from A travels inside the material to E with different $v_t$. If $v_t > v_i$, $v_t < v_i$ and $\overline{AE} < \overline{BD}$, the wavellet bends:

$$\frac{\sin \theta_i}{\overline{BD}} = \frac{\sin \theta_t}{\overline{AE}} \quad \Rightarrow \quad \frac{v_i}{v_t} = \frac{n_i}{n_t}$$

$$\frac{\sin \theta_i}{v_i \Delta t} = \frac{\sin \theta_t}{v_t \Delta t}$$

$\Rightarrow \quad n_i \sin \theta_i = n_t \sin \theta_t$ \quad Swell's law
Huygens principle holds in vacuum and materials.

**Vacuum**: Time-varying E and B field at point \( P \) causes spherical waves that move out from this point. (in a sense, each point of wavefront analogous to scattering center).

**Materials**: E.H wave impinging on dielectric material polarizes medium and drives electron oscillators into forced vibrations. These oscillators re-radiate or scatter energy in form of E.H waves at same \( r \) as incident waves. Waves from each oscillator superimpose to form secondary wave. Forced oscillation lags behind driving oscillation. Resultant or refracted wave lags the primary. Process is progressive one; as light traverses medium, it is continuously retarded in phase. Retardation increases with increasing light frequency, \( \gamma \) reason for dispersion \( n(\gamma) \) or \( n(\lambda) \).

**Electromagnetic Wave Theory / Light propagation**

- Gives specifications on relative intensities of reflected and refracted waves.
- Not fully treated here - subject of class on electro-magnetism; give outline of main physical arguments and expressions of results.
Ell theory shows that at boundary between two media, there exist certain restrictions for the $\vec{E}$ and $\vec{B}$ field of light.

Light wave hitting surface is reflected and refracted. Polarization usually arbitrary → taken apart into 2 components given by symmetry of arrangement. Plane of incidence defined by plane through incoming, refracted, and reflected beams.

$E_1$ to plane of incidence → parallel to refractive surf.

$E_2$ to plane of incidence → has components both $\parallel$ and $\perp$ to refractive surface (depending on entrance angle $\theta$: $\theta = 0$ → only $E_2$)

Ell theory shows: boundary conditions for $E$, $D = \varepsilon E$, $H$, and $B = \mu H$ at interface are different for field vectors $\parallel (t)$ or $\perp (n)$ to surface: certain quantities must be continuous through surface (i.e. must have same values on both sides), others not:

<table>
<thead>
<tr>
<th>Continuous</th>
<th>not continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_t$</td>
<td>$E_n$</td>
</tr>
<tr>
<td>$H_t$</td>
<td>$H_n$</td>
</tr>
<tr>
<td>$E_\parallel$</td>
<td>$E_\parallel$</td>
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<tr>
<td>$H_\parallel$</td>
<td>$H_\parallel$</td>
</tr>
<tr>
<td>$\mu H_t$</td>
<td>$\mu H_n$</td>
</tr>
<tr>
<td>$\mu H_\parallel$</td>
<td>$\mu H_\parallel$</td>
</tr>
</tbody>
</table>
Application of these continuity conditions first yields Snell's and reflection law. Also: setup continuity equations for all geometries and \( E, D, H, B \). This leads to Fresnel equations which give quantitative results about amplitude coefficients for reflection and transmission.

A. Derivation:

Consider (monochromatic) plane wave propagating towards planar surface which separates two isotropic media. Any \( \vec{E} \) or \( \vec{B} \) field can be treated in its projections \( \perp \) and \( \parallel \) to plane of incidence:

1. case \( \vec{E} \perp \) plane of incidence (\( \vec{B} \parallel \))

\[ \begin{align*}
\vec{E}_i & \quad \vec{E}_r \\
\theta & \quad \phi \\
\vec{E}_t & \quad \vec{H}_i \\
\phi & \quad \theta \\
\vec{H}_r & \quad \vec{H}_t
\end{align*} \]

\( \vec{K} \times \vec{E} = \nu \vec{B} \), \( \vec{K} \cdot \vec{E} = 0 \); \( E, B, k \) : right-handed system

continuity of tangential \( \vec{E} \) field across interface:

\[ (1) \quad \vec{E}_{0i} + \vec{E}_{or} = \vec{E}_{0t} \quad (\cos f = t \text{ cancels}) \]
Incoming wave with $E$ field normal to plane of incidence
continuity of \( \vec{B} \) field (normal to \( \vec{E} \)) across interface:

\[
(2) \quad - \frac{\vec{B}_i}{\mu_i} \cos \theta_i + \frac{\vec{B}_r}{\mu_r} \cos \theta_r = - \frac{\vec{B}_t}{\mu_t} \cos \theta_t
\]

(incasing \( x \)-direction is positive \( \Rightarrow \vec{B}_i, \vec{B}_t \) negative)

Since \( |\vec{B}_i| = \frac{|\vec{E}_i|}{v_i} \), \( |\vec{B}_r| = \frac{|\vec{E}_r|}{v_r} \), \( |\vec{B}_t| = \frac{|\vec{E}_t|}{v_t} \) and

Since \( v_i = v_r \) (same medium) and \( \theta_i = \theta_r \) (reflection):

\[
(2) \quad \frac{1}{\mu_i v_i} (|\vec{E}_i| - |\vec{E}_r|) \cos \theta_i = \frac{1}{\mu_t v_t} |\vec{E}_t| \cos \theta_t
\]

at \( \gamma = 0 \):

\[
\vec{E}_i = E_{0i} \cos (k_i r - \omega t) = E_{0i} \quad \hat{\text{e}}_r
\]

\[
\vec{E}_r = \vec{E}_{0r} \quad \ldots\ldots
\]

\[
\vec{E}_t = \vec{E}_{0t} \quad \ldots\ldots
\]

\[
(4) \quad \frac{n_i}{\mu_i} (|\vec{E}_{0i}| - |\vec{E}_{0r}|) \cos \theta_i = \frac{n_t}{\mu_t} |\vec{E}_{0t}| \cos \theta_t
\]

\[
|\vec{E}_{0i}| + |\vec{E}_{0r}| = |\vec{E}_{0t}|
\]
1. Set of Friedel equations (holding for any linear, isotropic, homogeneous medium):

\[
\left( \frac{\lvert \mathbf{E}_{ov} \rvert}{\lvert \mathbf{E}_{oi} \rvert} \right) = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}
\]

\[
\left( \frac{\lvert \mathbf{E}_{ot} \rvert}{\lvert \mathbf{E}_{oi} \rvert} \right) = \frac{2 n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}
\]

In case of dielectric media, for which \( n_i \neq n_t \neq n_0 \) (encountered very often):

amplitude coeff. for reflection (\( r_1 \))

\[
r_1 = \left( \frac{\lvert \mathbf{E}_{ov} \rvert}{\lvert \mathbf{E}_{oi} \rvert} \right) = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}
\]

amplitude coeff. for transmission:

\[
t_1 = \left( \frac{\lvert \mathbf{E}_{ot} \rvert}{\lvert \mathbf{E}_{oi} \rvert} \right) = \frac{2 n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}
\]
2. Case \( \mathbf{E} \parallel \) plane of incidence

(continuity of tangential components of \( \mathbf{E} \) across interface, and continuity of \( \mathbf{H} \) =>

2 additional Fresnel relations)

\[
\begin{align*}
| \mathbf{E}_{0\parallel} | \cos \theta_i - | \mathbf{E}_{0\perp} | \cos \theta_r &= | \mathbf{E}_{0\parallel} | \cos \theta_t \\
\frac{1}{\mu_i v_i} | \mathbf{E}_{0\parallel} | + \frac{1}{\mu_r v_r} | \mathbf{E}_{0\perp} | &= \frac{1}{\mu_t v_t} | \mathbf{E}_{0\parallel} |
\end{align*}
\]

with \( \mu_i = \mu_r \), \( \theta_i = \theta_r \) (reflection)

\[
T_{\parallel} = \left( \frac{| \mathbf{E}_{0\parallel} |}{| \mathbf{E}_{0\parallel} |} \right)_{\parallel} = \frac{\mu_t}{\mu_i} \frac{\cos \theta_i}{\cos \theta_t} - \frac{\mu_i}{\mu_t} \frac{\cos \theta_t}{\cos \theta_i}
\]

\[
T_{\perp} = \left( \frac{| \mathbf{E}_{0\perp} |}{| \mathbf{E}_{0\parallel} |} \right)_{\parallel} = 2 \frac{\mu_t}{\mu_i} \frac{\cos \theta_i}{\cos \theta_t} + \frac{\mu_i}{\mu_t} \frac{\cos \theta_t}{\cos \theta_i}
\]
Incoming wave with $E$ field in plane of incidence
for case of dielectric media ($\mu_i = \mu = \mu_0$)

\[
\tau_{II} = \frac{\eta_i \cos \theta_i - \eta_t \cos \theta_t}{\eta_i \cos \theta_i + \eta_t \cos \theta_t}
\]

\[
\tau_{II} = \frac{2\eta_i \cos \theta_i}{\eta_i \cos \theta_i + \eta_t \cos \theta_t}
\]

Introducing Snell's law ($\eta_i \sin \theta_i = \eta_t \sin \theta_t$) into

Fresnel's eqns., one obtains (for dielectrics):

\[
\tau_{II} = - \frac{\sin (\theta_i - \theta_t)}{\sin (\theta_i + \theta_t)}
\]

\[
\tau_{II} = \frac{\tan (\theta_i - \theta_t)}{\tan (\theta_i + \theta_t)}
\]

\[
\tau_{II} = \frac{2 \sin \theta_t \cos \theta_i}{\sin (\theta_i + \theta_t)}
\]

\[
\tau_{II} = \frac{2 \sin \theta_t \cos \theta_i}{\sin (\theta_i + \theta_t) \cos (\theta_i - \theta_t)}
\]

\text{Caution: Fresnel eqns. must be related to specific field directions, as defined earlier; other eqns. found in literature depend on different assumptions for direction of } \vec{E}_c \text{ and } \vec{B}_c. \]

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B. Interpretation of Fresnel Equations
   - Eqs. allow us to quantitatively determine relative amplitude coefficients of reflected and transmitted waves, and from these values, their intensity ratios.
   - allow us to obtain phase shifts in processes.

I. Amplitude coefficients

1. Special case: \( n_i < n_e \) (external reflection)
   a) \( \theta_i = 0 \): normal incidence
      \[
      r_\parallel = -r_\perp = \frac{n_e - n_i}{n_e + n_i}
      \]
      \( r \) for air/glass interface \( r_\parallel = -r_\perp = \frac{1.5 - 1}{1.5 + 1} = \pm 0.2 \)
      \( r \Rightarrow \text{intensity} \equiv r_\parallel^2 = \left( \frac{n_e - n_i}{n_e + n_i} \right)^2 = \left( \frac{0.5}{2.5} \right)^2 = 0.04 = 4\% \)
   b) \( \theta_i \neq 0 \)
      reflectivity increases with angle for \( \parallel \) component in steady fashion until it reaches 100% at grazing incidence \( (\theta_i = 90^\circ) \)
      since \( \theta_i > \theta_e \) for \( n_i < n_e \): \( r_\parallel = \frac{\sin(\theta_e - \theta_i)}{\sin(\theta_e + \theta_i)} < 0 \)
      \( \text{for } 0 \leq \theta_i \leq 90^\circ \)
however:
\[ T_\parallel > 0 \text{ for } 0 \leq \delta_p \leq \delta_p \]
\[ \delta_p = \text{polarization angle, a Brewster angle} \]
\[ T_\parallel < 0 \text{ for } \delta_p \leq \delta_c \leq 90^\circ \]
\[ T_\parallel = \left( \frac{\tan(\delta_c - \delta_p)}{\tan(\delta_c + \delta_p)} \right) = 0 \]
for \( \delta_c + \delta_p = 90^\circ \sqrt{V} \)
for Brewster angle:
\[ \frac{\sin \delta_i}{\sin \delta_c} = \frac{n_e}{n_c} = \frac{\sin \delta_c}{\sin(90^\circ - \delta_c)} - \frac{\sin \delta_c}{\cos \delta_c} \]
\[ \theta \text{ condition:} \]
\[ \tan \delta_p = \frac{n_e}{n_c} \quad (= 56^\circ \text{ for air/glass interface}) \]
Light reflected from interface at this angle has no component of E vector in plane of incidence
\( \Rightarrow \) all reflected light is linearly polarized \( \perp \) to plane of incidence
also: if incident light is already polarized \( \parallel \) to plane of incidence \( \Rightarrow \) total transmission into medium.
c) \( t_{ll} = t_{\perp} = \frac{\sin \gamma}{n_i + n_e} \) for \( \gamma = 0 \)

\( \text{air/glass: } t_{ll} = t_{\perp} = \frac{\sin \gamma}{2.5} = 0.8 \)

d) \( t_{ll} + t_{\perp} > 0 \) for \( 0 \leq \gamma \leq 90^\circ \)
always: \( t_{\perp} + (-t_{\perp}) = 1 \) for all \( \gamma \); 
but: \( t_{ll} + t_{\perp} = 1 \) only for \( \gamma = 0 \)

\( \text{can 2} \quad n_i > n_e \) (internal reflection)
\( \gamma_i > \gamma_e \)
\( \gamma_{ll} > \gamma_{\perp} \)
\( \Rightarrow \ t_{\perp} = -\frac{\sin (\gamma_i - \gamma_e)}{\sin (\gamma_i + \gamma_e)} > 0 \) for \( 0 \leq \gamma_i \leq \gamma_e \)
\( \gamma_c \equiv \text{critical angle for total internal reflection} \)
(angle at which \( t_{ll} \) reaches \( \pi/2 \))

\( t_{\perp} \) reaches \( +1 \) at \( \gamma_c \)
\( t_{\perp} = 1 \) for \( 0 \leq \gamma_i \leq 90^\circ \)
\( t_{ll} < 0 \) for \( 0 \leq \gamma_i \leq \gamma_e \)
\( t_{ll} > 0 \) for \( \gamma_e \leq \gamma_i \leq 90^\circ \)
\( t_{ll} = 1 \) for \( \gamma_e \leq \gamma_i \leq 90^\circ \)

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C. Phase shifts

Negative sign in Fresnel eqns. for \( \bar{n} \) means

\[ \Delta \Phi = \frac{\pi}{2} \Rightarrow E_{i1} \text{ and } E_{r1} \text{ will be anti-parallel upon reflection (out-of-plane by } \pi) \; \]

but \( t_{11} \) and \( t_{1} \) always parallel, no phase shift.

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To arbitrarily polarized light: decomposition into two components $E_\parallel$ and $E_\perp$. In reflection, each of them changes magnitude (and possibly sign) depending on $\theta_i$ and $n_1 > n_2$; after reflection, vector addition of reflected component $E_\parallel$ and $E_\perp$ yields polarization + amplitude of reflected wave.
Coefficients for the power of the reflected (R) and transmitted light (T).

\[
\begin{align*}
\text{Intensity of wave, } I \quad (\text{photon flux density}) & : \\
I & = \frac{N \cdot h \nu \sim n E_0^2}{\text{area \cdot time}} \\
\text{Power of light, } P \quad (\text{photon flux}) & : \\
P & = \text{Intensity \cdot area} \sim n E_0^2 \cdot \text{area} \\
\text{Reflectance, } R & = \frac{\text{Reflected Power}}{\text{Incident Power}} = \frac{n_i \cos \theta_r \cdot E_r^2}{n_i \cos \theta_i \cdot E_i^2} = r^2 \\
\text{Transmittance, } T & = \frac{\text{Transmitted Power}}{\text{Incident Power}} = \frac{n_i \cos \theta_i \cdot E_i^2}{n_i \cos \theta_i \cdot E_i^2} = \frac{n_i \cos \theta_i}{n \cos \theta_i} \cdot f^2
\end{align*}
\]
Total energy flowing into area A per unit time equals energy flowing outward from A per unit time:

\[ I_\nu A \cos \vartheta = I_\nu A \cos \vartheta + I_\nu A \cos \vartheta \]

or

\[ n_c E_0 \cos \vartheta = n_c E_0 \cos \vartheta + n_t E_0 \cos \vartheta \]

Same conservation holds for \( R \) and \( T \) components:

\[ R_{\parallel} + T_{\parallel} = 1 \]
\[ R_{\perp} + T_{\perp} = 1 \]

When \( \vartheta = 0 \) (normal incidence), distinction between \( \parallel \) and \( \perp \) of \( R, T \) disappears:

\[ R = R_{\parallel} = R_{\perp} = \left( \frac{n_t - n_c}{n_t + n_c} \right)^2 \]
\[ T = T_{\parallel} = T_{\perp} = \frac{4 n_t n_c}{(n_t + n_c)^2} \]

Air-glass interface \( (n_t = 1; n_c = 1.5) \):

\[ R = r^2 = \left( \frac{n_t - 1}{n_t + 1} \right)^2 = 0.04 \quad \text{\( \{ \) then \( R + T = 1 \) \}
\[ T = n_t^2 = \frac{4 n_t}{(n_t + 1)^2} = 0.96 \]

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To 4% of light incident normally on air-glass interface will be reflected back (internally or externally).

Reflectance and transmittance vs incident angle $\theta_i$.

Plot of $R$ at single interface (normal incidence) for transparent media with increasing refractive index $n_i$. Example: Reflection from diamond surface ($n = 2.52$) : $R \approx 25\%$.
D. Total internal reflection / evanescent waves

When \( \theta_i < \theta_c \) critical, all incoming energy is reflected back into incident medium (total internal reflection, TIR, \( \theta_c \approx 42^\circ \) for glass/air interface).

At interface, for \( \theta_i = \theta_c \), a surface wave emerges.

(If one assumes there is no such wave, it becomes impossible to satisfy boundary conditions). Surface wave, called evanescent wave, has special property that, on average, cannot carry energy through interface.

**Derivation**:

\[
\begin{align*}
\mathbf{T}_{\parallel} &= \left( \frac{E_{0v}}{E_{0i}} \right)_{\parallel} = \frac{n_c \cos \theta_i - n_k \cos \theta_k}{n_c \cos \theta_i + n_k \cos \theta_k} \\
&= \frac{\cos \theta_i - \left( \frac{n_k^2}{n_c^2} \sin^2 \theta_i \right)^{1/2}}{\cos \theta_i + \left( \frac{n_k^2}{n_c^2} \sin^2 \theta_i \right)^{1/2}} \\
\mathbf{T}_{\perp} &= \left( \frac{E_{0v}}{E_{0i}} \right)_{\perp} = \frac{n_k^2 \cos \theta_i - (n_k^2 - \sin^2 \theta_i)^{1/2}}{n_k^2 \cos \theta_i + (n_k^2 - \sin^2 \theta_i)^{1/2}} \\
&= -165
\end{align*}
\]
Since $\sin \theta_c = n_{ci}$ for $\lambda_i = \lambda_c$

$\sin \theta_c > n_{ci}$ (*) for $\lambda_i > \lambda_c$

$\Rightarrow \Gamma$ in above expressions -- negative

$\Rightarrow T_{II}, T_{II}$ are complex quantities.

Inspite of this, $T_{II}^{*} + T_{II} \cdot T_{II}^{*} = 1$, and $\Re = 1$

which means that $I_r = I_i$, and $I_t = 0$ (no energy across boundary).

Wave function for transmitted $E$ field:

$\hat{E}_t = \hat{E}_{0t} e^{i(k_r \hat{r} - \omega t)}$

\[ \hat{K}_t \cdot \hat{r} = k_{tx} x + k_{ty} y \]

\[ k_t \cos \theta_t = \pm k_t \left(1 - \frac{\sin^2 \theta_t}{n_{ci}^2} \right)^{1/2} \]

Snell's law

Since $\sin \theta_t > n_{ci}$ :
\[ k_{tx} = \frac{k_t}{\sin \gamma_i} \]

Shell's law

\[ E_t = E_{0t} \exp \left( \frac{i}{\hbar} \left( k_{tx} \sin \gamma_i - \omega t \right) \right) \]

evanescent wave amplitude drops off exponentially inside the less dense medium (only a few wavelengths of penetration) tends in +x direction.

conservation of energy: wave circulates back and forth across interface resulting, on average, in zero net flow through boundary into second medium.

exponential decay of evanescent wave was confirmed experimentally at optical frequencies (\( \gamma \)) by concept of frustrated total internal reflection

use as beam splitter

(uses low-index transparent film as precision spacer)

analogous to barrier penetration a tunneling in quantum mechanics
E. Optical Properties of Metals

- in dielectrics, conductivity \( \sigma = 0 \) j in metals \( \sigma \neq 0 \) electrons in conduction band can be driven into oscillations j electrons collide to conversion of EM energy into heat \( \rightarrow \) absorption j
- effects can be described with complex permittivity \( \epsilon = \epsilon_R - i\epsilon_I \)

\( \epsilon \) e.m. wave is written as

\[ \vec{E} = \vec{E}_0 \cos \omega (t - \frac{y}{c}) \quad \text{\( \tilde{\omega} \) complex} \]

\[ \vec{E} = \vec{E}_0 e^{-i\omega y/c} \cos \omega (t - \frac{\omega y}{c}) \]

travels in \( y \)-dir. with \( \frac{\epsilon_R}{\epsilon_I} \)

attenuated towards inside of conductor by this term

\[ I(y) = I_0 e^{-2y} \text{ with } \alpha = \frac{2\omega \epsilon_I}{c} \]

(absorption or attenuation coefficient)

\[ y = \frac{1}{\alpha} \text{ known as skin or penetration depth} \]

\( \sim 100 \text{ Å in copper at visible } \lambda \)}
For metals to be transparent, thickness must be smaller than penetration depth, thin films.

Reflectivity, $R$, of metals calculated from reflect. coeff., $j$

for $\theta = 0$ (normal incidence): $R = \frac{n_e - n_i}{n_e + n_i}$

$\Rightarrow R = \frac{I_r}{I_i} = R_{||} R_{||}^*$

$= \left(\frac{\tilde{n}_e - 1}{\tilde{n}_e + 1}\right) \left(\frac{\tilde{n}_i - 1}{\tilde{n}_i + 1}\right)^*$

for air ($n_i = 1$) / metal ($n_e = \tilde{n}$) interface

with $\tilde{n} = n_e - c n_i$.

$R = \frac{(n_e - 1)^2 + n_i^2}{(n_e + 1)^2 + n_i^2}$

Examples:

<table>
<thead>
<tr>
<th>Metal</th>
<th>$n_e$</th>
<th>$n_i$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tin</td>
<td>1.5</td>
<td>5.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Solid sodium</td>
<td>0.04</td>
<td>2.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Gold</td>
<td>0.04</td>
<td>2.4</td>
<td>0.25</td>
</tr>
<tr>
<td>Silver</td>
<td>0.04</td>
<td>2.4</td>
<td>0.97</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.04</td>
<td>2.4</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(for 589.3 nm light)