

**Light Polarization**

A. So far: discussed only linearly polarized light (the usual case). Now: discuss other states of polarization, and how to observe it, produce it, and make use of it. EM wave theory can describe phenomena very well.

Considers two orthogonal waves, superimposed, having same frequency:

1. \( \mathbf{E}_x(z, t) = \mathbf{\hat{e}} E_{0x} \cos (kz - \omega t) \)

2. \( \mathbf{E}_y(z, t) = \mathbf{\hat{e}} E_{0y} \cos (kz - \omega t + \delta) \quad \delta \) is phase angle

\[ \delta = \mathbf{\hat{e}} E_{0y} \cos \left[ \Delta x \left( \frac{2\pi}{\lambda} - \omega t \right) \right] \]

\( \Rightarrow \) spatial shift between both waves

Resulting wave is vector sum:

\[ \mathbf{E}(z, t) = \mathbf{E}_x(z, t) + \mathbf{E}_y(z, t) \]

**Important cases**:

1. \( \delta = 0 \) or \( \delta = \pm \pi \) \( \Rightarrow \) waves are in phase

\( \text{spatial shift } 0, \lambda, 2\lambda \)

\[ \mathbf{E} = (\mathbf{\hat{e}} E_{0x} + \mathbf{\hat{e}} E_{0y}) \cos (kz - \omega t) \]

(resultant wave has fixed amplitude (in plane of wave)

\( \Rightarrow \) it is linearly polarized

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2. $E$ is odd integer multiple of $\pm \pi$:

$$E = \pm m\pi \quad \text{where } m = 1, 3, 5, \ldots$$

waves are $180^\circ$ out of phase

$$E_E = (i E_{0x} - j E_{0y}) \cos (kx - wt)$$

wave again linearly polarized but plane of vibration has rotated
3. \( E_{ox} = E_{oy} = E_0 \) \( \text{and} \) \( \varepsilon = -\frac{\pi}{2} + 2\pi n \)
\( m = 0, \pm 1, \pm 2, \ldots \)

(or: spatial shift of \( \lambda/4, \; 3\lambda/4, \; \ldots, \; 2\pi m + \frac{\lambda}{4} \))

13. \( \vec{E}_x (z,t) = i \; E_0 \cos (Kz - \omega t) \)
\( \vec{E}_y (z,t) = j \; E_0 \sin (Kz - \omega t) \)

\[ \vec{E} = E_0 \left[ i \cos (\cdot) + j \sin (\cdot) \right] \]

direction of \( \vec{E} \) is now time-varying, and is not restricted to single plane
4. more general case: magnitude of $E$ changes as well as direction:

(i) $E_x = E_{ox} \cos (k_z - \omega t)$

(ii) $E_y = E_{oy} \cos (k_z - \omega t + \varepsilon)$

(iii) $\frac{E_y}{E_{oy}} = \cos (k_z - \omega t) \cos \varepsilon - \sin (k_z - \omega t) \sin \varepsilon$

(iv) $\sin (k_z - \omega t) = \sqrt{1 - \left(\frac{E_x}{E_{ox}}\right)^2}$

(v) \[ \left(\frac{E_y}{E_{oy}} - \frac{E_x}{E_{ox}} \cos \varepsilon\right)^2 = \left[1 - \left(\frac{E_x}{E_{ox}}\right)^2\right] \sin^2 \varepsilon \]

or

\[ \left(\frac{E_y}{E_{oy}}\right)^2 + \left(\frac{E_x}{E_{ox}}\right)^2 - 2\left(\frac{E_x}{E_{ox}}\right)\left(\frac{E_y}{E_{oy}}\right) \cos \varepsilon = \sin^2 \varepsilon \]

eqn. of ellipse, having angle $\alpha$ with respect to $E_x, E_y$ coordinate system

with $\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos \varepsilon}{E_{ox}^2 + E_{oy}^2}$
Summary

(a) $E_x$ leads $E_y$ by:
- $2\pi$ to $7\pi/4$
- $3\pi/2$ to $5\pi/4$
- $\pi$ to $3\pi/4$
- $3\pi/2$ to $5\pi/4$
- $\pi/2$ to $\pi/4$
- $0$

(b) Diagram showing the relationship between $E_x$ and $E_y$ with angles $2\pi$, $3\pi/2$, $\pi$, $3\pi/2$, and $2\pi$.
Any polarization direction can always be described by $E_x, E_y$ components in the perpendicular directions.

Natural light = rapidly varying succession of different polarization states emitted by atoms ($\sim 10^8$ excited state lifetime) = "randomly polarized". All polarization components $\rightarrow$ to propagation direction. Polarizer transmits only $E$ along one direction $\rightarrow$ produces linearly polarized light.

A second polarizer ('analyzer') with angle $\phi$ against first polarizer will transmit amplitude

$$E = E_0 \cos \phi$$

and intensity

$$I = I_0 \cos^2 \phi$$

Malus's Law
Light polarization can be realized by reflection, dichroism, scattering and birefringence.

3. Light reflection

Reflection of light at dielectric surface under Brewster angle → produces 100% polarized light. Only 4%, however, reflected at glass surface. Improved by "piles of plates" polarizer.

C. Dichroism

Property of certain materials (tourmaline crystals) to absorb light of different polarization in a different way. This results in different color (= dichroism) for differently polarized light in these crystals.
Today: dichroic polarizer sheets: Polaroid

Historical note: Herapath ( ) analyzed small, green, needle-shaped crystals under microscope (crystals formed in urine of dog, which had been fed with quinine (!!!), after adding drops of iodine ). Observed remarkable effect: when needles overlapped, either dark when crossed perpendicular ( ) or transparent when crossed under small angle ( ). This "quinine sulfate periodide" (= "herapathite") first observed dichroic crystals. Land (1928) read about this, ground "herapathite" into submicroscopic needle-shaped crystals, aligned them first by E or B fields - later colloidal suspension of needle crystals extended through long narrow slit → 'Polaroid J-sheets' (large effective dichroic crystal, but problems with scattering, hazy appearance).

Modern versions: sheets of clear polyvinyl alcohol heated and stretched: its long hydrocarbon molecules become aligned. Iodine impregnated: iodine molecules attach to long chain polymer molecules and form chain of their own.
Components of \( \vec{E} \) in incident light wave which are parallel to molecule chain, drive electrons, become absorbed. Components of \( \vec{E} \bot \) to chain are transmitted.

**D. Light scattering**

Light waves impinging on atom, molecule excites oscillating electric dipole and leads to re-radiation of electromagnetic energy at same frequency. The closer frequency of exciting light to atomic eigensfrequency, the more light is scattered. Most atoms, molecules eigensfrequency in UV \( \rightarrow \) light scattering increases from red to blue light. Quantitatively for objects of size \( d \ll \lambda \) : Rayleigh scattering

\[
S \sim \frac{1}{\lambda^4}
\]

*Example: blue sky*

\( \text{O}_2 \) and \( \text{N}_2 \) molecules of atmosphere scatter (absorb and re-emit) mostly blue light. Sunset: red light left over! Clouds: large particles (ice crystals in cirrus) \( \lambda \) \( \rightarrow \) reflection and refraction effects don't depend much on \( \lambda \).
Point out: If particles regularly arranged (like lattice) → scattered waves will interfere such that only constant part of incoming wave results → no attenuation from scattering: Huygens' principle.

Conclusion: scattering only from density and refractive index fluctuations and inhomogeneities. These are large for low density gas — small for pure liquids and even smaller for perfect crystals.

Polarization + scattering: linearly polarized $\vec{E}$ field drives oscillation of dipole in same direction $\rightarrow$ emits dipole radiation, polarized like driving $\vec{E}$ field. No radiation.
in dipole direction. No light observed when watched

\[ \downarrow \] \text{to incoming light beam in E direction!} \]

\[ \text{If incoming light unpolarized} \rightarrow \text{superposition of both} \]
\[ \text{polarized pictures. Observation} \]
\[ \downarrow \text{to incoming beam K will} \]
\[ \text{show only light polarization} \]
\[ \text{E} \parallel K. \text{Therefore skylight} \]
\[ \text{is partially polarized — best} \]
\[ \text{observed in direction} \downarrow \text{to} \]
\[ \text{incoming sun light.} \]
\[ \text{Multiple scattering in atmosphere prevents full polarization} \]
\[ \text{effect of light. Some insects orient by polarized} \]
\[ \text{skylight. Human eye does not have built-in polarizer} \]
\[ \text{(nature might have given it up since it was not needed).} \]
E. Birefringence (Double Refraction)

So far: homogenous, isotropic materials (glass, motten quartz) with isotropic refractive index. Crystals (not cubic ones, like NaCl) of low symmetry structure can have anisotropic optical properties.

Birefringence faces of electron oscillator different in different directions — different response (polarizability) to light wave in \( x, y, \) or \( z \) direction \( \Rightarrow \) different refractive indices \( n_x, n_y, n_z \)

Either \( n_z \neq n_x = n_y \) uniaxial crystal \( z \equiv \) optical axis

or \( n_z \neq n_x \neq n_y \) biaxial crystal

Birefringence related to symmetry of crystal structure — example calcite (calcium carbonate, \( \text{CaCO}_3 \); builds marble, limestone) structure: optical axis. All carbonate groups are in planes \( \perp \) to optical axis \( z \)

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each CO$_3$ group forms triangular cluster whose plane is \( \perp \) to optical axis and whose symmetry is 3-fold.

When \( \mathbf{E} \) is in plane \( \perp \) to optical axis:
- optically isotropic \( n_x = n_y \).

When \( \mathbf{E} \) is in plane containing optical axis \( z \):
- \( n_z \neq n_x, n_y \).

If a light beam traverses calcite crystal normal to a cleavage plane, it splits and separates as two parallel beams \( \rightarrow \) 2 images \( \square \)

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When rotating crystal, one image remains stationary: 'ordinary' rays; the other one rotates: 'extraordinary' rays. Both images are linearly polarized and orthogonally polarized to each other. Reason: double refraction.

Calcite rhombohedron:
parallel sides and edges: optic axis makes equal angles with edges at blunt corners (where edges meet at angles of 102°)

principal section (parallel to optic axis and also perpendicular to pair of opposite cleavage surfaces)
Each principal section is parallelogram with angles 109° and 71°. If incident wave is linearly polarized \( \perp \) to optical axis and propagates through principal section \( \perp \) to its surface \( \rightarrow \) no refraction, ordinary (straight line) propagation \( \perp \) through material.

If incident wave linearly polarized \( \parallel \) to principal section, \( E \) has component \( \parallel \) to optical axis \( \rightarrow \) reflection (propagation along optical axis with \( v_{\parallel} \neq v \)).

For calcite: \( v_{\perp} = \frac{c}{1.658} \) and \( v_{\parallel} = \frac{c}{1.486} \) (for yellow light).

\( \Rightarrow v_{\perp} < v_{\parallel} \)
Since \( v_\parallel > v_\perp \), wavelets inside crystal elongate in all directions normal to optical axis (ellipsoids of revolution around optical axis). Envelope of all ellipsoidal wavelets is portion of plane wave parallel to incident wave, but plane waves displaced sideways when traveling through calcite crystal. \( \Rightarrow \) In anisotropic crystal, direction of ray is not normal to wavefront:

\[
\mathbf{S} = \mathbf{v} \times \mathbf{E} \times \mathbf{B}
\]

\( \uparrow \) tensor

\[
\mathbf{D} = \varepsilon \mathbf{E} \equiv \text{displacement vector}
\]
F. Birefringent Polarizers

Uniaxial crystals with birefringence can be used to make linear polarizer: famous example: Nicol prism

Rhombohedra cut into 2 halves, cemented with Canada balsam ($n = 1.55$). Ordinary ray propagating through prism will be totally internally reflected at calcite-balsam interface; extraordinary ray will be transmitted → polarization

Similar: Glan-Thompson prism (polarizer)

2 calcite prisms; optic axis II; incoming ray I to surface. Total internal reflection at interface for one polarization (o-ray) passes e-ray: $n_e < \frac{1}{\sin \theta} < n_o$; 10° field of view; transparent over large wavelength range 280 nm → 5 µm
6. Circular and Elliptical Polarization

In certain media, circular or elliptical polarization can be used to change the polarization state of an incoming wave. In this function, they are termed retarders or retardation plates, meaning that one of the two constituent polarization states making up the incoming wave is caused to lag in-phase behind the other by a predetermined amount. This makes it possible to convert any given polarization state into any other and to sense, e.g., circular and elliptical polarization!

1) If \( \hat{E} \)-vector is \( \parallel \) or \( \perp \) to optical axis (i.e., along direction of principal refractive index):
   - Linear polarized light remains linear polarized, but
   - Travels with different phase velocities \( \frac{\nu_x}{\nu_y} \).

2) If \( \hat{E} \)-vector forms angle \( \theta \) with
   - \( n_x \), \( n_y \) directions (optical axis):
   - Decompose into 2 components \( E_x, E_y \) which are initially in plane. After travelling through
birefringent medium of thickness $d$:

$$E_y^t = E_0 \sin \frac{2\pi}{\lambda} \left( \frac{d \cdot n_y}{\lambda} - vt \right)$$

$$E_x^t = E_0 \sin \frac{2\pi}{\lambda} \left( \frac{d \cdot n_x}{\lambda} - vt \right)$$

After traversing birefringent plate ($n_x \neq n_y$; thickness $d$), the two waves $E_y$ and $E_x$ have again same wavelength $\lambda$, but are shifted relative to each other by

$$\Delta \phi = \frac{2\pi d}{\lambda} (n_x - n_y)$$

or

$$\Delta = d (n_x - n_y)$$

special cases:
<table>
<thead>
<tr>
<th>Phase angle shift</th>
<th>Spatial shift</th>
<th>Retardation plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \psi$ (retardation)</td>
<td>$\Delta \text{retardation}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2}, \frac{3\pi}{2}, \ldots, \frac{(2m+1)\pi}{2}$</td>
<td>$\frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots, \frac{(2m+1)\lambda}{4}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2}, \frac{3\pi}{2}, \ldots, (2m+1)\pi$</td>
<td>$\frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots, \frac{(2m+1)\lambda}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

Conditions for material with indices $n_1, n_2$ to construct quarter-wave plate for wavelength $\lambda$:

$$(n_1 - n_2) d = \frac{2m+1}{4} \lambda$$

$$d = \left(\frac{2m+1}{4}\right) \lambda \frac{1}{n_1 - n_2}$$

$\frac{\lambda}{4}$ (quarter-wave) plate for $\lambda$ is half-wave plate for half the wavelength.
Optic axis

Detail - within crystal

Timable retarder (Soleil-Babinet compensator):
2 moveable wedges and plane-parallel slab of birefringent materials crossed.

\[ d_1 \text{ - variable} \Rightarrow \text{variable retardation effect} \]
II. Optical Activity

Certain materials introduce continuous rotation of plane of polarization for light which propagates through medium (right or left). Convention: look against direction of travelling light: clockwise = dextro-rotary; counter clockwise = levo-rotary.

Example: quartz: 2 different crystallographic structures (d + l): same molecules (SiO₂) but different arrangement (one mirror image of other). Right or left handed structure.

![Diagram of quartz crystals showing right and left handed structures](image)

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Phenomenological reason for optical activity: medium has two different refractive indices for right and left circularly polarized light ("circular birefringence").

At \( z = 0 \) (fixed observation point):

![Diagram](image)

A linearly polarized \( \vec{E} \) field can be regarded as a superposition of two oppositely rotating circular fields:

- **Right circular wave**: \( \vec{E}_r = E_0/\sqrt{2} \begin{bmatrix} \cos (k_2 z - \omega t) + j \sin (k_2 z - \omega t) \end{bmatrix} \)
- **Left circular wave**: \( \vec{E}_l = E_0/\sqrt{2} \begin{bmatrix} \cos (k_2 z - \omega t) - j \sin (k_2 z - \omega t) \end{bmatrix} \)

Then \( \vec{E}_{\text{lin}} \bigg|_{k_2 = k_0} = \vec{E}_r + \vec{E}_l = E_0 \begin{bmatrix} \cos (k_2 z - \omega t) \end{bmatrix} \)

Since \( \omega = \text{const} \), \( k_2 = k_0 n_2 \) and \( k_L = k_0 n_L \). So, if \( n_2 \neq n_L \) \((n_2 \neq n_L)\), right circular polarized wave will travel with different speed compared to left circular wave. After both waves passed through medium, vector addition of \( \vec{E}_r \) and \( \vec{E}_l \) forms again linearly polarized \( \vec{E} \).
but rotated count - clockwise or clockwise:

\[ \hat{E} = \hat{E}_L + \hat{E}_R = E_0 \cos \left[ \left( k_2 - k_L \right) \frac{z}{d} - \omega t \right] \times \]

\[ \left[ \hat{i} \cos \left( k_L - k_2 \right) \frac{z}{d} + \hat{j} \sin \left( k_L - k_2 \right) \frac{z}{d} \right] \]

At \( z = 0 \) point, where wave enters medium, wave is polarized along \( x - \) axis: \( \hat{E} = E_0 \hat{i} \cos \omega t \) at any point along path inside medium, both components of \( \hat{E} \) have the same time dependence and are therefore in phase \( \Rightarrow \) linearly polarized, but orientation of linear \( \hat{E} \) is function of \( z \). If \( n_2 > n_L \), then \( k_2 > k_L \) and \( \hat{E} \) rotates count - clockwise (looking into source) i.e. field at point \( z \) makes angle \( \beta = -(k_2 - k_L) \frac{z}{d} \) with respect to original orientation.

For medium of thickness \( d \):

\[ \beta = \frac{2 \pi d \left( n_L - n_2 \right)}{\lambda} \]

- \( n_2 > n_L \): \( d \) - rotatory
- \( n_2 < n_L \): \( L \) - rotatory

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Circular birefringence not only in crystals (property of the crystalline building structure) — but occurs also for many organic materials (like sugar) in solution; i.e. rotary power must be property of single molecule.

Exact calculation of effect very complicated (requires quantum mechanics). However, physical essence of how effect is produced can be explained and illustrated with simplified model, visualizing the optically active molecule as a left- or right-handed spiral structure, with the \( \hat{E} \) field of light \( \perp \) to its axis. The light \( \hat{E} \) field will introduce an electric momentum \( \hat{p} \) and the current in the spiral a magnetic momentum \( \hat{m} \). Depending on whether spiral is left- or right-handed, \( \hat{p} \) and \( \hat{m} \) are either parallel or anti-parallel. Both \( \hat{p} \) and \( \hat{m} \) dipoles \( \pi \)-radial e.m. waves. \( \hat{E} \)-field emitted by electric moment \( \hat{p} \) \( (E_p) \) is \( \perp \) to \( E_m \) emitted by magnetic dipole — vector addition; resulting field \( E_\ell \) scattered by helix

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not II to incident field $E_i$. Resultant transmitted light (vector sum of $E_i + E_s$) is rotated in direction determined by rotational sense of helix. Amount of rotation depends on angle of helix axis to $E_i$ — but has always the same direction for helices of same sense. Analogy simplified but works: linearly polarized microwaves ($\lambda = 3\text{ cm}$) incident on box of identical copper helices (1 cm long, insulated from each other) $\to$ transmitted wave shows rotation of plane of incidence.

Fascinating observations concerning optical activity in field of biology: many organic molecules occur in two forms which are mirror images (stereo-isomers), but chemically identical; 'd- and l-isomers', right or left rotating.

When organic molecules synthesized in laboratory: equal amounts d and l occur, always. One would expect equal d and l amounts in natural organic substances, but not the case at all! Sugar, e.g., ($\text{CH}_2\text{O}_6$) wherever grown, from cane or beets, always d-rotatory. Simple sugar dextrose 'd-glucose' ($\text{C}_6\text{H}_{12}\text{O}_6$), most important carbohydrate in human metabolism, always d. So living organisms distinguish between optical isomers.

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All proteins fabricated from amino acids (made from C, O, N, H). There are twenty-odd amino acids, all (but the simplest which is non-isomeric) are L-rotatory: protein molecule, broken up from any living material on earth, will show the constituent amino acid to be L-rotatory. (One important exception: group of antibiotics, like penicillin, contains dextrorotatory amino acids — may account for toxic effect of penicillin on bacteria). Speculation about origin of life on this and other planets: did life on earth originally consist of both mirror-image forms? Amino acids found in meteorites and in lunar samples. So far, apparently equal amounts of L and D amino acids mark contrast to overwhelmingly predominance of left-handed forms in terrestrial rocks.
Field of Stress-Induced Polarisation Effects

So far, polarisation properties (birefringence or optical activity) are properties of the intrinsic structure of the material (arrangement and binding of atoms). These properties can also be externally induced:

a) Stress-optical effects or photo-elasticity or mechanical birefringence: under compression or tension material takes on properties of negative or positive uniaxial crystal. Transparent isotropic materials become birefringent under stress. Direction of stress → optical axis, birefringence → stress. Non-uniform stress → non-uniform birefringence or retardation effects.

Test of transparent objects (lenses etc.) between crossed polarizers reveal internal stress (improper annealing). Transparent scale-models of part to be studied made from stress-optically sensitive materials (epoxy, resin,...) subjected to model forces: complicated fringe pattern reveals internal stresses. Retardation by stress depends on wavelength ($\lambda_1$, $\lambda_2$, $\lambda$ retardation) – is chromatic fringes show regions of equal stress. Superimposed on colored fringes is different system of
black bands: $E_{\parallel}$ to stress - no change in polarisation for light of all wavelengths $\Rightarrow$ black between crossed polarizers (isodichroic bands).

b) Electro-optical effects

Isotropic transparent material becomes birefringent in $E$-field: $n_{\parallel} E \neq n_{\perp} E$; called "Kerr effect", after John Kerr (1875).

\[ \Delta n = n_{\parallel} - n_{\perp} = \lambda \cdot k \cdot E^2 \]

$k = \text{Kerr constant}$.

Kerr effect $\sim E^2$ therefore called \underline{quadratic electro-optic effect}. Mostly found in liquids - electric-field alignment of optically anisotropic molecules. Possible in solids, too.
Kerr constants for polar liquids:

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Kerr constant</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>benzene</td>
<td>$0.6 \times 10^{-7}$ cm stat volt$^{-2}$</td>
<td></td>
</tr>
<tr>
<td>carbon disulfide</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>water</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>nitrobenzene</td>
<td>220</td>
<td>$\leftarrow$ explosive, poisonous</td>
</tr>
</tbody>
</table>

Example for Kerr-based on Kerr–effect: Kerr shutter polar liquid between crossed polarizers $\rightarrow$ no voltage $\Rightarrow$ no transmission $\rightarrow$ finite voltage $\Rightarrow$ variable waveplate $\rightarrow$ amplitude modulation $\rightarrow$ high frequency range: $10^{10}$ GHz

Retardation: $\Delta \varphi = 2\pi K \ell V^2 / d^2$

$\ell \sim 30$ kV for $\pi$ plate of nitrobenzene

Electrode distance $d = 1$ cm, and $\ell \approx$ several cm

More attractive (lower voltages, non-hazardous): special mixed crystals displaying linear electro-optic or Pockels effect: $\Delta n \sim E$.

Works with crystals having no inversion symmetry (no central point through which every atom can be reflected into identical atom): crystals like KDP (potassium dihydrogen phosphate). KDP: modulation frequencies up to 25 GHz; $\sim 7$ kV for $\pi$ retardation at 540 nm (5 times less voltage than equiv. liquid cells)
c) Faraday effect or Magneto-optical effect

Light through material which is placed in strong magnetic field \( (\parallel \text{ to light propagation}) \) rotates plane of light polarization. Rotation angle (empirical):

\[
\beta = \nu B d
\]

\[\nu \equiv \text{Vernon constant}\]

Example:

Water, \( d = 1 \text{ cm} \),

\( B = 10^4 \text{ Gauss} \)

\( \Rightarrow \beta \approx 2^\circ \)

Sign of Vernon constant indicates direction of rotation (+ for \( \beta \)-rotatory with light \( \parallel \vec{B} \) field \( \beta \)-for \( \beta \)-rotatory with light \( \perp \vec{B} \).

Phenomenologically: magnetic field induces circular birefringence (explanation: quantum mechanics - Zeeman effect: splitting of energy states of electron into magnetic components)

Magneto-optical effect used in Faraday modulator: signal to be transmitted modulates \( \vec{B} \); emerging laser beam is amplitude modulated, since Faraday rotation depends on axial
component of magnetization to current controls $\beta$.

d) **Light Modulation and Switching**

Stress-, Electro-, and Magneto-optical effects used in light modulation and switching. Material placed between crossed or parallel polarizers (i.e., zero or max. transmission). Applied $E$, $H$, or $S$ produces birefringence and retardation effects: light transmission increases or decreases with applied field. Example: Kerr or Pockels cell between crossed polarizers — no light transmission. Electric field "switches on" the light. Modulated $E$ modulates light. Response time of Kerr or Pockels effect determines highest modulation frequency ($\sim 10^{10}$ Hz for Kerr cell; $10^{8}$ Hz for Pockels cell). Application of shutters in high-speed photography; Q-switches of pulsed lasers. Similar:

Faded-out effect — potential for light modulator to impress information on laser beam (by changing amplitude, polarization direction, phase ... of light in manner related to signal to be transmitted.)
Faraday effect: use of excitonic ferromagnetic materials (e.g., Yttrium–iron garnet (YIG) doped with gallium). Magnetized to saturation with transversal H field. Modulated field in coil introduces magnetization component in longitudinal direction as Faraday effect—a coil current. Analyser converts polarization rotation $\beta(H)$ into amplitude or intensity modulation via Malis' law.

Stress-optical modulator: material stress-modulated by piezoelectric transducers (choose modulation frequency = eigenfrequency of material: large effect)

1) Electric Field (E)
   a) Kerr Effect $(\Delta n \propto E^2)$
      \[
      \Delta n = n_r - n_l = \lambda_0 \kappa E^2
      \]
      Retardation:
      \[
      \Delta \varphi = \frac{2\pi}{\lambda} \Delta n \cdot d
      \]
      (nitrobenzene)
      \[E=30000 \text{V/cm}, d=5 \text{cm} \rightarrow \Delta \varphi=\text{r}\
   
   b) Pockels Effect
      \[
      \Delta \varphi \propto \Delta n \propto E
      \]
      Light
      Example: "KDP" $(KH_2PO_4)$ crystals

2) Magnetic Field (H)
   "Faraday Effect"
   \[
   \Delta n = n_r - n_l
   \]
   And therefore Rotation of linearly polarized light. $\beta \propto H \cdot d$
   Example: Water $d=1 \text{cm}$
   $H=10^4 \text{ Gauss} \rightarrow \beta = 2 \text{r}\]

3) Stress Effect (Photo-elasticity)
   Stress $S$ produces linear birefringence
   $n_r - n_l = \Delta n \propto S$
K. Light Polarisation in Photon Picture

Photons carry energy \((E, \nu)\), linear \((\frac{E}{c})\) and angular \((\pm \frac{h}{2\pi})\) momentum.

Pure circularly polarized light: stream of photons with identical (left or right) spin. If falling on target and absorbed: angular momentum conserved \(\Rightarrow\) exerts torque on material; can be measured (circularly polarized \(E\)-field drives electrons in material into circular orbits).

If wave reflected at dielectric interface \(\Rightarrow\) Fresnel signs.

\[
\text{Fresnel equation reflection (small angle): one } r\text{-coeff. is positive, one negative.}
\]

One \(E\) component is phase-shifted by \(\chi_\nu\) or \(180^\circ\).

\[
\begin{array}{c|cc}
& r_{\|} & r_{\perp} \\
\hline
r_{\perp} & + & \nu_i < \nu_t \\
r_{\|} & - & \nu_i > \nu_t \\
\end{array}
\]

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Linear momentum vector inverted in reflection; angular momentum vector remains the same. Light changes from right to left circular (relative to propagation vector) — but no change in L: no transfer of angular momentum.

Linear polarized light — superposition of equal amounts of right and circular polarized light with specific phase.

Stream of linear polarized quanta as same amount of well-defined + and - photons. Not right: photons all identical; exist simultaneously in both spin states with equal likelihood: - + equal + +. Beam will not impart angular momentum to target.

If not in both spin states with same probability (+ + more often than - +) no net angular momentum imparted.

—follow-up remarks on polarisation—

1) **Natural light** ( unpolarized): 2 orthogonally polarized waves of equal amplitude with randomly varying phase (within ~ 10^-8 sec according to lifetime of excited atomic state).

2) **Circular polarized light**: 2 orthogonally polarized waves of equal amplitude, with fixed phase difference (\( \frac{\pi}{4} \), \( \frac{3\pi}{4} \lambda \), ...)
Both show no intensity change if linear polarization is turned in light beam. But if variable retarder is introduced: circular light can be converted (by compensating or changing the phase shift to 0 or $\pi/2$) into linear polarized light.

3) Clarification about convention for direction of circularly polarized light

a) Wave fixed in time: wave along propagation vector $\vec{k}$ of wave: $\vec{E}$ vector describes right- or left-handed helix $\Rightarrow$ this you call right- or left circularly polarized light, respectively.

b) Wave fixed in space (fixed $xy$ plane). Look against propagation vector $\vec{k}$ of light and observe time variation of $\vec{E}$ vector in fixed $xy$ plane $\Rightarrow$ defines right- or left circular motion.

Both definitions (a) and (b) are identical $\checkmark$
4. Circularly polarized light reflected from dielectric surface:

- **Incoming wave** (right circular)

- **Reflected wave**; experienced 1/2 phase shift in one component (Fresnel). Wave changes from right to left circular

- Angular momentum unchanged.

- **Incorrect**: Reflected wave without phase shift would remain right circular, but rotation direction in space changed.

- Angular momentum changed.