Lecture 13: June 19th 2009

Physics for Scientists and Engineers II
Ampere’s Law

Magnetic field lines are tangential to a circle surrounding the current.

\[ \oint B \cdot ds = B \oint ds = \frac{\mu_0 I}{2\pi r} 2\pi r = \mu_0 I \]
Ampere’s Law

\[ \oint B \cdot ds = \int_{left\ semicircle} B \cdot ds + \int_{straight\ section} B \cdot ds + \int_{right\ semicircle} B \cdot ds + \int_{straight\ section} B \cdot ds \]

Picking a different path.

Top View
Ampere’s Law

\[ \oint \vec{B} \cdot d\vec{s} = \int_{\text{left semicircle}} \vec{B} \cdot d\vec{s} + \int_{\text{straight section}} \vec{B} \cdot d\vec{s} + \int_{\text{right semicircle}} \vec{B} \cdot d\vec{s} + \int_{\text{straight section}} \vec{B} \cdot d\vec{s} \]

\[ = \frac{\mu_0 I}{2\pi r_1} \int_0^\pi \vec{n} \cdot d\vec{s} + 0 + \frac{\mu_0 I}{2\pi r_2} \int_0^\pi \vec{n} \cdot d\vec{s} + 0 \]

\[ = \frac{\mu_0 I}{2} + \frac{\mu_0 I}{2} = \mu_0 I \]

The line integral \( \oint \vec{B} \cdot d\vec{s} \) around any closed path equals \( \mu_0 I \), where I is the total steady current passing through any surface bounded by the closed path.

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I \]
Ampere’s Law

\[ \oint B \cdot ds = \int_{\text{inner semicircle}} B \cdot ds + \int_{\text{straight section}} B \cdot ds + \int_{\text{right semicircle}} B \cdot ds + \int_{\text{straight section}} B \cdot ds \]
Ampere’s Law

\[ \oint \vec{B} \cdot d\vec{s} = \int_{\text{inner semicircle}} \vec{B} \cdot d\vec{s} + \int_{\text{straight section}} \vec{B} \cdot d\vec{s} + \int_{\text{right semicircle}} \vec{B} \cdot d\vec{s} + \int_{\text{straight section}} \vec{B} \cdot d\vec{s} \]

\[ = -\frac{\mu_0 I}{2\pi r_1} \int_0^{\pi r_1} ds + 0 + \frac{\mu_0 I}{2\pi r_2} \int_0^{\pi r_2} ds + 0 \]

\[ = -\frac{\mu_0 I}{2} + \frac{\mu_0 I}{2} = 0 \]

The line integral \( \oint \vec{B} \cdot d\vec{s} \) around any closed path equals \( \mu_0 I \), where I is the total steady current passing through any surface bounded by the closed path.

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I \]
Example Application of Ampere’s Law

Inner conductor: \( r < r_1 \)

\[
\oint B \cdot d\vec{s} = \mu_0 I_{\text{inside}} \Rightarrow B 2\pi r = \mu_0 \frac{I_1}{\pi r_1^2} \pi r^2 \Rightarrow B = \mu_0 \frac{I_1}{2\pi r_1^2} r
\]

Insulator (air): \( r_1 < r < r_2 \)

\[
\oint B \cdot d\vec{s} = \mu_0 I_{\text{inside}} \Rightarrow B 2\pi r = \mu_0 I_1 \Rightarrow B = \mu_0 \frac{I_1}{2\pi r_1}
\]
Example Application of Ampere’s Law

Outer conductor: \( r_2 < r < r_3 \)

\[
\oint B \cdot d\vec{s} = \mu_0 I_{\text{inside}} \Rightarrow B \ 2\pi \ r = \mu_0 \left( I_1 - I_2 \frac{\pi r_1^2 - \pi r_2^2}{\pi r_3^2 - \pi r_2^2} \right)
\]

\[
\Rightarrow B = \frac{\mu_0}{2\pi r} \left( I_1 - I_2 \frac{\pi r_1^2 - \pi r_2^2}{\pi r_3^2 - \pi r_2^2} \right)
\]

Outside: \( r_3 < r \)

\[
\oint B \cdot d\vec{s} = \mu_0 I_{\text{inside}} \Rightarrow B \ 2\pi \ r = \mu_0 (I_1 - I_2)
\]

\[
\Rightarrow B = \frac{\mu_0 (I_1 - I_2)}{2\pi r}
\]
\[ \oint B \cdot ds = 0 \Rightarrow \int B \cdot ds = 0 \quad \text{but not necessarily that } \vec{B} = 0 \]

In fact, we know that \( \vec{B} \neq 0 \) along the blue path. Rather, \( \vec{B} \) is perpendicular to the path (coming out of the page).
\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I \] along the red path.

\[ \Rightarrow \] Conclusion: You already need to know from symmetry etc. the direction of the magnetic field and that the magnitude of $B = \text{constant}$ along the path to draw the right conclusions from Ampere's laws.
A Long Solenoid (Wire wound in the form of a helix)

Long solenoids produce reasonably uniform magnetic fields in their "interior".

\[ \oint \vec{B} \cdot d\vec{s} = 0 \]

Doesn't mean that \( B = 0 \)!!!!

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I \]

\( \Rightarrow \vec{B} \neq 0 \) outside the coil, but it is small.
A Long Solenoid (Wire wound in the form of a helix)

Make the assumption that $B_{\text{external}} \ll B_{\text{internal}}$

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 N I \]

$(N = \text{number of turns inside the blue loop})$

\[ \Rightarrow B L = \mu_0 N I \quad \Rightarrow \quad B = \mu_0 \frac{N}{L} I = \mu_0 n I \]

$(n = \frac{N}{L} \quad \text{is the number of turns per unit length})$
Problem 33 in the book

\( J_s \) (out of paper in y-direction) is a linear current density.

Show that \( \vec{B} \) is parallel to the sheet, perpendicular to \( J_s \) and \( B = \mu_0 \frac{J_s}{2} \).
Problem 33 in the book

\( J_s \) (out of paper in \( y \)-direction)
is a linear current density.

Symmetry shows that \( \mathbf{B} \) is parallel to the sheet and perpendicular to \( J_s \).

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I
\]

\[
\Rightarrow 2B L = \mu_0 I \quad \Rightarrow \quad B = \mu_0 \frac{1}{2} \frac{I}{L} = \frac{1}{2} \mu_0 J_s
\]
Gauss’s Law in Magnetism

Magnetic flux though surface element d\(\vec{A}\) : \(d\Phi_B = \vec{B} \cdot d\vec{A}\)

Magnetic flux though a surface : \(\Phi_B \equiv \int \vec{B} \cdot d\vec{A}\)

Units of magnetic flux : \(1 \text{Tm}^2 = 1\text{Wb}\) (weber).

Gauss's law in magnetism : The magnetic flux through any closed surface is always zero :

\[\oint \vec{B} \cdot d\vec{A} = 0\]
Gauss’s Law Comparison

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{inside}}}{\varepsilon_0} \]

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]

Electric flux through closed surface is proportional to the amount of electric charge inside (electric monopoles).

Isolated magnetic monopoles have never been found.
Magnetism in Matter

We now know how to build “electromagnets” (using electric current through a wire). We also found that a simple current loop produces a magnetic field / has a magnetic dipole moment.

How about the “current” produced by an electron running around a nucleus? Let’s use a classical model (electron is a point charge orbiting around a positively charged nucleus.)

Orbital angular momentum of electron

The tiny current loop produces a magnetic moment
Magnetism in Matter

\[ I = \frac{q}{t} = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{e v}{2\pi r} \]

\[ \mu = IA = \left( \frac{e v}{2\pi r} \right) \pi r^2 = \frac{1}{2} e v r \]

\[ L = m_e v r \quad \Rightarrow \quad v r = \frac{L}{m_e} \]

\[ \Rightarrow \mu = \frac{1}{2} e \frac{L}{m_e} = \left( \frac{e}{2m_e} \right) L \quad (\mu \propto L) \]

\( L = \text{“orbital angular momentum”} \)
Quantum Physics: Orbital angular momentum L is "quantized"

(means: it only occurs in multiples of \( \hbar = \frac{\hbar}{2\pi} = 1.05 \times 10^{-34} \text{ Js} \) \( \hbar \) = Plank's constant).

⇒ smallest nonzero value of \( \mu \):

\[
\mu = \left( \frac{e}{2m_e} \right) L = \sqrt{2} \frac{e}{2m_e} \hbar
\]

(the factor \( \sqrt{2} \) has to do with "total" orbital angular momentum, which is different from that only in one direction....this is beyond the scope of this class).
Spin

Spin: An intrinsic property of electrons that contributes to the total magnetic moment of an electron.

Magnitude of spin angular momentum: \( S = \frac{\sqrt{3}}{2} \hbar \)

Magnetic moment due to spin: \( \mu_{\text{spin}} = \frac{e \hbar}{2m_e} = 9.27 \times 10^{-24} \, \frac{J}{T} \)

\( \frac{e \hbar}{2m_e} = \mu_B \) is called the Bohr magneton

\[ \vec{\mu}_{\text{total}} = \vec{\mu}_{\text{orbital}} + \vec{\mu}_{\text{spin}} \]

For atoms where all electrons are "paired" \( \Rightarrow \vec{\mu}_{\text{spin}} = 0 \)

For atoms with uneven number of electrons an unpaired electron must exist \( \Rightarrow \vec{\mu}_{\text{spin}} \neq 0 \)

The nucleus of an atom also has a magnetic moment due to it's protons and electrons.

\( \mu_{\text{nucleus}} \ll \mu_{\text{electron}} \)