Lecture 20: July 10th  2009

Physics for Scientists and Engineers II
Transmission line economics

Consumer runs vacuum cleaner (1KW power)

\[ P_{\text{consumer}} = I \Delta V_C = I^2 R_L \quad \Rightarrow \quad I^2 = \frac{P_{\text{consumer}}}{R_L} \]

\[ P_{\text{lost}} = I^2 R_T = \frac{P_{\text{consumer}}}{R_L} R_T \]

\[ P_{\text{generator}} = I^2 (R_T + R_L) = \frac{P_{\text{consumer}}}{R_L} (R_T + R_L) = P_{\text{consumer}} \frac{R_T}{R_L} + P_{\text{consumer}} = P_{\text{lost}} + P_{\text{consumer}} \]
Transmission line economics – using transformers

\[ I_G = \frac{N_2}{N_1} I_T = \frac{N_2}{N_1} \frac{N_1}{N_2} I = I \]

\[ I_T = \frac{N_1}{N_2} I \]

\[ R_T \]

\[ R_L \]

Step-up transformer \( N_1 < N_2 \)

\[ P_{\text{consumer}} = I^2 R_L \implies I^2 = \frac{P_{\text{consumer}}}{R_L} \]

\[ P_{\text{lost}} = I_T^2 R_T = \left( \frac{N_1}{N_2} I \right)^2 R_T = \left( \frac{N_1}{N_2} \right)^2 I^2 R_T = \left( \frac{N_1}{N_2} \right)^2 \frac{P_{\text{consumer}}}{R_L} R_T \]

\[ P_{\text{generator}} = I^2 R_{\text{eq}} = \frac{P_{\text{consumer}}}{R_L} \left( \left( \frac{N_1}{N_2} \right)^2 R_T + \left( \frac{N_1}{N_2} \right)^2 \left( \frac{N_2}{N_1} \right)^2 R_L \right) = \frac{P_{\text{consumer}}}{R_L} \left( \left( \frac{N_1}{N_2} \right)^2 R_T + R_L \right) = P_{\text{consumer}} \left( \frac{N_1}{N_2} \right)^2 \frac{R_T}{R_L} + P_{\text{consumer}} = P_{\text{lost}} + P_{\text{consumer}} \]
Example: Problem 43 in Chapter 33

Transmission line has resistance per unit length : \( 4.50 \times 10^{-4} \ \Omega/m \)

Length of transmission line : 400 miles (6.44 \times 10^5 m)

Need to transmit 5.00MW of power

\( V_{\text{generator}} = 4.50kV \) transformed up to 500KV

What is the loss of power in the transmission line?

\( R_T = 4.50 \times 10^{-4} \ \Omega \times 6.44 \times 10^5 m = 2.90 \times 10^2 \ \Omega \)

\( I_G = \frac{P_{\text{transmitted}}}{V_G} = \frac{5.00MW}{4.50kV} = 1.11 \times 10^3 A \)

\( I_T = \frac{N_1}{N_2} I_G = \frac{V_1}{V_2} I_G = \frac{4.50kV}{500kV} 1.11 \times 10^3 A = 10.0 A \)

\( P_{\text{lost}} = I_T^2 R_T = (10.0 A)^2 \times 2.90 \times 10^2 \Omega = 29.0 kW \)

\( \frac{P_{\text{lost}}}{P_{\text{transmitted}}} = \frac{29.0 \times 10^3}{5.00 \times 10^6} = 0.00580 = 0.580\% \)
Example: Problem 43 in Chapter 33

Without transformer:

\[ R_T = 4.50 \times 10^{-4} \frac{\Omega}{m} \times 6.44 \times 10^5 m = 2.90 \times 10^2 \Omega \]

\[ I_G = \frac{P_{\text{transmitted}}}{V_G} = \frac{5.00 MW}{4.50 kV} = 1.11 \times 10^3 A \]

\[ I_T = I_G = 1.11 \times 10^3 A \]

\[ P_{\text{lost}} = I_T^2 R_T = (1.11 \times 10^3 A)^2 \times 2.90 \times 10^2 \Omega = 358 MW > P_{\text{Generator}} \]

⇒ This cannot be. How much can be transmitted?
Example: Problem 43 in Chapter 33

\[ I_{\text{max}} = \frac{\Delta V}{R_L + R_T} \]

\[ P_{\text{load}} = I_{\text{max}}^2 R_L = \left( \frac{\Delta V}{R_L + R_T} \right)^2 R_L = \Delta V^2 \frac{R_L}{R_L^2 + 2R_T R_L + R_T^2} \]

Find power maximum:

\[ \frac{dP_{\text{load}}}{dR_L} = \Delta V^2 \frac{R_L^2 + 2R_T R_L + R_T^2 - R_L \left(2R_L + 2R_T\right)}{\left(R_L^2 + 2R_T R_L + R_T^2\right)^2} \equiv 0 \]

\[ \Rightarrow R_L^2 + 2R_T R_L + R_T^2 - R_L \left(2R_L + 2R_T\right) = 0 \]

\[ \Rightarrow R_T^2 - R_L^2 = 0 \quad \Rightarrow \quad R_L = R_T \]

\[ P_{\text{load max}} = I_{\text{max}}^2 R_L = \left( \frac{4.50kV}{R_L + 290\Omega} \right)^2 R_L = \left( \frac{4.50kV}{580\Omega} \right)^2 290\Omega = 17.5kW \]

(and this is ONLY when \( R_L = R_T \). In all other cases even less power can be transmitted). BTW, same principle as with internal resistance of a battery, where we found that the maximum power is transferred to load when \( R_L = R_{\text{internal}} \).
The 500000 Volt Tesla Coil – featuring Adam Beehler successfully trying to not electrocute himself
Remember Ampere's law?

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]

where \( I = \frac{dq}{dt} \) is a conduction current (created by charge carriers).

Problem: I only passes through \( S_1 \) but both \( S_1 \) and \( S_2 \) are bound by the same path. However, while a conduction current passes through \( S_1 \) none passes through \( S_2 \).
General Form of Ampere’s Law

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]

is clearly inadequate for this situation.

James Clark Maxell: Generalized Ampere's law to include time-varying electric fields. He defined a displacement current:

\[ I_d \equiv \varepsilon_0 \frac{d\Phi_E}{dt} \]

where \( \varepsilon_0 = \) permittivity of free space.

and \( \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} \) is the electric flux.

General form of Ampere's Law
(Ampere-Maxwell Law):

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Calculating displacement current: Example

Surface $S_1$: $\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + 0) = \mu_0 I$

Surface $S_2$: $\oint \vec{B} \cdot d\vec{s} = \mu_0 (0 + I_d) = \mu_0 \varepsilon_0 \frac{d \Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d}{dt} EA$

$= \mu_0 \varepsilon_0 \frac{d}{dt} \frac{q}{\varepsilon_0} = \mu_0 \frac{dq}{dt} = \mu_0 I$
Major Point

General form of Ampere's Law
(Ampere - Maxwell Law):
\[ \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

Magnetic fields are produced both by conduction currents and by time-varying electric fields.
Example: Chapter 34, Problem 1

A 0.100 - A current is charging a capacitor that has square plates 5.00 cm on each side. The plate separation is 4.00 mm. Find a) the time rate of change of electric flux between the plates and b) the displacement current between the plates.

Plate Capacitor: \( E = \frac{q}{\varepsilon_0 A} \rightarrow \Phi_E = EA = \frac{q}{\varepsilon_0} \)

\[
\frac{d\Phi_E}{dt} = \frac{dq}{\varepsilon_0} = \frac{1}{\varepsilon_0} \frac{d}{dt} = \frac{1}{\varepsilon_0} I = \frac{0.100 A}{8.854 \times 10^{-12} \frac{C^2}{Nm^2}} = 1.13 \times 10^{10} \frac{Nm^2}{Cs}
\]

(b) \( I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{1}{\varepsilon_0} I = 0.100 A \)
A Complete Description of All Classical Electromagnetic Interactions in a Vacuum

Maxwell's Equations in free space

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \quad \text{(Gauss's law)} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{(Gauss's law in magnetism)} \]

\[ \oint \mathbf{E} \cdot ds = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)} \]

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Ampere - Maxwell law)} \]

\[ \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad \text{(Lorentz force law)} \]
Maxwell’s Prediction

Maxwell's Equations in free space when $q = 0$ and $I = 0$

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 0 \quad \text{(Gauss's law)} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \text{(Gauss's law in magnetism)} \]

\[ \int \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law)} \]

\[ \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Ampere-Maxwell law)} \]

Can be combined $\Rightarrow$ Result: Wave equations for electric and magnetic fields

Solution of wave equations shows that electric and magnetic field waves travel ("electromagnetic radiation") with the speed of light, leading Maxwell to predict that light is a form of electromagnetic radiation.
Hertz’s Experiment

Hertz showed with this apparatus that electromagnetic waves exist.
Hertz’s Experiment

Details of the apparatus:

How it works: Battery charges the capacitor in the primary circuit on the left. The interrupter periodically creates a short circuit across that capacitor and alternating charging and discharging of capacitor occurs. A large periodic change of flux occurs in the primary winding. The secondary winding picks up this large flux change and transforms the primary voltage fluctuation into a much higher voltage fluctuation in the secondary circuit. The secondary circuit is tuned (LC) creating a very large voltage across the spark balls. → High electric fields occur between the spark balls, causing a sudden charge transfer (spark). → The electric field breaks down but is then built up again. This fluctuating electric field induces a fluctuating magnetic field (Ampere-Maxwell law), which in turn creates a fluctuating electric field (Faraday’s law)…..→ electromagnetic radiation, which is then picked up by the detection ring causing it to spark across it’s spark gaps (detection ring is also tuned to the same frequency).
Hertz’s Experiment

Other experiments by Hertz:

"em wave generator"

Standing "em wave"

metal plate reflects "em wave."

Nodes can be detected $\Rightarrow$ wavelength can be determined

Since frequency is know, the speed of the em wave can be calculated using $\nu = \lambda f$. The result was $\nu = c$, the speed of visible light.
Plane versus Spherical Waves

Let’s first look at mechanical waves:
Imagine looking down on a pond. Someone throws rocks into the middle of the pond, creating **spherical waves** in two dimensions.

![Diagram of spherical waves](image)

Direction of wave propagation

If instead, the person throws very long metal bars into the pond, **plane waves** are created.

![Diagram of plane waves](image)

Metal bar

All parts of the wave front have the same phase.
Plane linearly polarized electromagnetic waves with propagation in x-direction.

Properties:
1) Wherever in the yz plane em wave comes from, it propagates in the x-direction.
   (all “rays” of this wave are parallel).
2) All em waves coming from different parts of the yz plane are all in phase.
3) Electric and magnetic waves oscillate perpendicular to each other (take this by faith for now).
4) Electric field is in the same direction (here in y-direction) $\Rightarrow$ “linearly polarized”.
5) Magnetic field is in the same direction (here in z-direction) $\Rightarrow$ “linearly polarized”.
6) And B are only function of x and t and do not depend on y and z $\Rightarrow$ E(x,t), B(x,t).