Lecture 22: July 15th 2009

Physics for Scientists and Engineers II
Momentum and Radiation Pressure

Electromagnetic radiation carries energy $T_{ER}$ and momentum $p$ per unit time. If $T_{ER}$ is totally absorbed by a surface perpendicular to $\vec{S}$, the momentum $\vec{p}$ transported to the surface has a magnitude:

$$p = \frac{T_{ER}}{c} \quad \text{(complete absorption)}$$

$\Rightarrow$ A pressure $P$ is exerted on the surface by the radiation ("radiation pressure")

$$P = \frac{F}{A} = \frac{1}{A} \frac{dp}{dt} = \frac{1}{A} \frac{d}{dt} \left( \frac{T_{ER}}{c} \right) = \frac{1}{c} \frac{dT_{ER}}{dt} = \frac{S_{\text{avg}}}{c} \frac{I}{c} \quad \text{(complete absorption)}$$

(Note: $P$ here is pressure, not power)

For complete reflection: $p = \frac{2T_{ER}}{c}$ and $P = \frac{2S_{\text{avg}}}{c} = \frac{2I}{c}$
Antennas

Stationary charges: Produce constant electric fields
Constant currents: Produce constant magnetic fields
Maxwell's equations tell us that we need time-varying fields to produce electromagnetic radiation.

⇒ Accelerated charges needed to produce time-varying fields.
Whenever a charged particle accelerates, it radiates energy (em radiation).

The Half Wave Antenna

Source of alternating voltage having frequency \( f = \frac{c}{\lambda} \)

Conducting rods
Driving the resonance frequency of the antenna ----???

You can create a driven oscillation by applying an alternating voltage with a frequency equal to the self-oscillation frequency of the antenna \( f = \frac{c}{2L} \).

\[ \Rightarrow \text{Resonance occurs with higher amplitudes (greater electric fields, greater current).} \]

Snapshot in time:

Separation of charges creates an electric field like that of a dipole:

\[ \vec{B} \perp \vec{E} \text{ at all points in space and time.} \]

\[ \Rightarrow \text{Given the name “Dipole antenna”} \]
Driving the resonance frequency of the antenna

Snapshot in time:

\[ \vec{S} \] (indicating the direction in which energy flows)

\[ \vec{B} \perp \vec{E} \] at all points in space and time.
\[ \vec{E} \] and \[ \vec{B} \] are 90° out of phase (e.g., when \( I = 0 \), the charges at the outer ends of the rods are at a maximum).
Voltage and Current Distribution
Driving the resonance frequency of the antenna
The “near field” (dipole field) behavior

Shortly thereafter, the current reverses:
\[ \Rightarrow \text{Poynting vector reverses direction} \]
\[ \Rightarrow \text{Energy flowing inward} \]
\[ \Rightarrow \text{no net flow of energy} \text{??? No! The dipole field (near field)} \]
\[ \text{falls of proportional to } \frac{1}{r^3} \text{ and quickly becomes insignificant.} \]
The “far field” behavior

At larger distances time-varying electric fields induce time-varying magnetic field and vice versa according to

\[ \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \]  
(Faraday's law)

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]  
(Ampere-Maxwell law)

The magnetic and electric fields produced in this manner are in phase.

E and B are proportional to \( \frac{1}{r} \)

\[ \Rightarrow \text{radiation looses intensity proportional to} \quad \frac{1}{r^2} \]
Angular dependence of radiation intensity

Intensity of Radiation: \[ I = S_{\text{avg}} \propto \frac{\sin^2 \theta}{r^2} \]

Similarly, when using an antenna to receive radiation, orientation matters equally.
Angular dependence of radiation intensity

Similarly, when using an antenna to receive radiation, orientation matters equally.

Good reception

No reception

No reception

\( \vec{E} \) should be parallel to antenna
Electromagnetic Spectrum

The Electromagnetic Spectrum

<table>
<thead>
<tr>
<th>Wavelength (metres)</th>
<th>Radio</th>
<th>Microwave</th>
<th>Infrared</th>
<th>Visible</th>
<th>Ultraviolet</th>
<th>X-Ray</th>
<th>Gamma Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^3$</td>
<td>$10^{-2}$</td>
<td>$10^{-5}$</td>
<td>$10^{-6}$</td>
<td>$10^{-8}$</td>
<td>$10^{-10}$</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$10^4$</th>
<th>$10^8$</th>
<th>$10^{12}$</th>
<th>$10^{15}$</th>
<th>$10^{16}$</th>
<th>$10^{18}$</th>
<th>$10^{20}$</th>
</tr>
</thead>
</table>

Physics for Scientists and Engineers II, Summer Semester 2009
The Nature of Light

Newton: Light = Stream of Particles (can explain reflection and refraction)

1678 Christian Huygens: Showed that a wave model of light can also explain reflection and refraction.

1801 Thomas Young: Experimental demonstration of the wave nature of light (Double slit experiment → “Interference” effects)

Maxwell: Light = high frequency electromagnetic wave

Hertz: Confirms existence of electromagnetic waves
…..but also discovers the photoelectric effect (contradicts the wave model, which predicts: more light intensity = more energy transferred to electrons)

Einstein 1905: Proposes that energy of light wave is present in particles called photons. $E=hf$ is the energy of a photon (explains photoelectric effect).
Measuring the speed of light

Galileo: Using lanterns between mountains didn’t work. The light is too fast.

1675: Ole Roemer found that the period of revolution of the moon Io around Jupiter depends on whether the earth is receding from or moving towards Jupiter (longer period when earth is receding). From these data Huygens calculated $c > 2.3 \times 10^8$ m/s

1849: Fizeau’s method (toothed wheel): $c = 3.1 \times 10^8$ m/s

Currently accepted value: $c = 2.9979 \times 10^8$ m/s
Eclipse Time Delay of Io in the shadow of Jupiter

Roemer’s method

In this position, the time of the eclipse is delayed by many minutes compared to the opposite position of the earth.

\[ c \approx \frac{2 \times R_{\text{sun-earth}}}{t_{\text{delay}}} \]

Physics for Scientists and Engineers II, Summer Semester 2009
Geometrical Optics

The study of the propagation of light under these assumptions:
1) In a uniform medium light travels in a straight line.
2) Light changes direction in a medium with non-uniform optical properties, or at the interface between two media.

The “Ray approximation”: Wave moving through a medium travels in a straight line in the direction of it’s rays.
Ray Approximation in Geometric Optics

Encountering barriers (like an opening in a wall):

The ray approximation is valid if \( \lambda \ll d \), where \( d \) is the size of the opening.

Wave continues in a straight line

\[ \lambda \ll d \]

Diffraction occurs

Opening acts like a point source of a wave.

\[ \lambda \gg d \]
Reflection of Light/Wave

\[ \theta'_1 = \theta_1 \]

Angle of reflection = Angle of incidence
Specular and Diffuse Reflection

Specular reflection
“Smooth surface”
Variations in surface $<< \lambda$

Diffuse reflection
The Wave under Refraction

Incident Ray

Normal

Reflected Ray

Air

Glass

Angle of Refraction

Refracted Ray

\[ \theta_1 > \theta_2 \]

\[ \theta_1' \]

\[ \nu_2 < \nu_1 \]

\[ \nu_1 \]
The Wave under Refraction

Incident Ray, Reflected Ray, and Refracted ray are all in one plane.

\[
\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \frac{\text{Speed of light in medium 2}}{\text{Speed of light in medium 1}}
\]
The Wave under Refraction

- Incident Ray
- Normal
- Reflected Ray
- Glass
- Air

- $\theta_1$: Incident Angle
- $\theta_1'$: Reflected Angle
- $\theta_2$: Refracted Angle
- $v_1$: Speed in Glass
- $v_2$: Speed in Air

$\theta_2 > \theta_1$
$v_2 > v_1$
Index of Refraction

Index of Refraction: \[ n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \]

For vacuum: \[ n_{\text{vacuum}} = 1 \]

All other media: \[ n > 1 \]

When light enters a medium:

- frequency remains constant, but wavelength changes.
- \( f_1 = f_2 = f \)
- \( v_1 = \lambda_1 f \)
- \( v_2 = \lambda_2 f \)

\[ \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\frac{v_1}{f}}{\frac{v_2}{f}} = \frac{v_1}{v_2} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1} \quad \Rightarrow \quad \lambda_1 n_1 = \lambda_2 n_2 \]

\[ \Rightarrow \quad n = \frac{\lambda}{\lambda_n} \quad \text{Wavelength in vacuum} \quad \lambda_n < \lambda \]

Wavelength in medium
Snell’s Law of Refraction – **not yet covered in class but needed for HW 16.**

\[
\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2}
\]

\[\Rightarrow \quad n_1 \sin \theta_1 = n_2 \sin \theta_2\]

*Snell’s Law of Refraction*