Lecture 28: August 3rd 2009

Physics for Scientists and Engineers II
The Single Slit Diffraction Pattern (Fraunhofer Diffraction Patterns)
Huygens Principle: Each part of the slit is a source of light waves.

A minimum: \( \frac{a}{2} \sin \theta = \pm \frac{\lambda}{2} \)

\[ \Rightarrow \sin \theta = \pm \frac{\lambda}{a} \]

Under these conditions: Ray 1 interferes destructively with 1b, 2 with 2b, etc…
The Single Slit Diffraction Minima

Divide slit into 4 equal parts:

A minimum : \( \frac{a}{4} \sin \theta = \pm \frac{\lambda}{2} \quad \Rightarrow \quad \sin \theta = \pm \frac{2\lambda}{a} \)

Divide slit into 6 equal parts:

A minimum : \( \frac{a}{6} \sin \theta = \pm \frac{\lambda}{2} \quad \Rightarrow \quad \sin \theta = \pm \frac{3\lambda}{a} \)

Divide slit into 2m equal parts:

A minimum : \( \frac{a}{2m} \sin \theta = \pm \frac{\lambda}{2} \quad \Rightarrow \quad \sin \theta = \pm m \frac{\lambda}{a} \)

\[
\sin \theta_{dark} = m \frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3 \ldots ...
\]
Intensity Distribution of the Single-Slit Diffraction Pattern

\[ I = I_{\text{max}} \left( \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\pi a \sin \frac{\theta}{\lambda}} \right)^2 \]

\[ I = 0 \quad \text{when} \quad \frac{\pi a \sin \theta_{\text{dark}}}{\lambda} = m \pi \]

⇒ \[ \sin \theta_{\text{dark}} = m \frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots \]
Intensity Distribution of the Single-Slit Diffraction Pattern

\[ I = I_{\text{max}} \left( \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\pi a \sin \frac{\theta}{\lambda}} \right)^2 \]

where

- \( I \) is the intensity distribution,
- \( I_{\text{max}} \) is the maximum intensity,
- \( a \) is the slit width,
- \( \theta \) is the angle of observation,
- \( \lambda \) is the wavelength of the light,
- \( \sin \left( \frac{\pi a \sin \theta}{\lambda} \right) \) is the modulation amplitude,
- \( \pi a \sin \frac{\theta}{\lambda} \) is the phase difference.
Intensity Distribution of the Double-Slit Diffraction Pattern

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \left[ \sin \left( \frac{\pi a \sin \theta}{\lambda} \right) \right]^2 \]

Double slit pattern – as discussed before

Each of the two slits exhibits the single slit diffraction pattern
Double Slit Diffraction Pattern (with single slit envelope)
\[ \frac{d}{a}=5 \]

Intensity Distribution of the Double-Slit Diffraction Pattern

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \left[ \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\frac{\pi a \sin \theta}{\lambda}} \right]^2 \]
Suppressed Maxima in the Double Slit Pattern

\[ I = I_{\text{max}} \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right) \]

Maxima for \( d \sin \theta = m\lambda \)

Minima for \( a \sin \theta = n\lambda \quad n = \pm 1, \pm 2 \ldots \)

\[
\frac{d \sin \theta}{a \sin \theta} = \frac{m \lambda}{n \lambda} \implies \frac{m}{n} = \frac{d}{a}
\]

⇒ For these ratios of m and n the double slit maximum is erased because it falls on a single slit minimum.

Example : \( n = 1 \) (first minimum of single slit) for \( \frac{d}{a} = 5 \) erases the double slit maximum at \( m = \frac{d}{a} = 5 \) (See previous page).
Resolution of Single Slit Apertures

Just barely resolved center maxima (max. of S1 falls on first min. of S2. (Rayleigh’s criterion)

Unresolved center maxima for smaller angle.
Resolution of Single Slit Apertures

Just barely resolved center maxima (max. of S1 falls on first min. of S2. (Rayleigh’s criterion)

Location of first minimum: \( \sin \theta = \frac{\lambda}{a} \)

Limiting angle of resolution for a slit of width \( a \):

\[ \theta_{\text{min}} \approx \frac{\lambda}{a} \quad \text{(approx. for small angles)} \]
Resolution of Single Slit Apertures

Limiting angle of resolution for a round aperture of diameter \( D \):

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D}
\]

Example: Telescopes using mirrors of a certain diameter:

\( \rightarrow \) The larger the mirror, the smaller the limiting angle of resolution.

(Critical in viewing close objects that are far away).
Maxima occur at the same angles as with double slit:

\[ d \sin \theta_{\text{bright}} = m \lambda \quad m = 0, \pm 1, \pm 2, \ldots. \]

However: Maxima a sharper, minima are wider
\[ \Rightarrow \text{better ability to resolve different wavelength s.} \]
The “Diffraction” (Interference) Grating

\[ d \sin \theta_{\text{bright}} = m \lambda \quad m = 0, \pm 1, \pm 2, \ldots \]
Analyze the spectral lines of atoms (learn about electronic energy levels). Identify the composition of gases (e.g., what atoms are sending out light from a distant star (send light through spectrometer and look for “fingerprint” of atoms.
X-Ray Diffraction

Wavelength of x-rays: $\lambda \approx 0.1\text{nm}$

$\Rightarrow$ For a reasonable angle (e.g., $10^\circ$) of a maximum:

$$d = \frac{\lambda}{\sin 10^\circ} \approx \frac{0.1 \times 10^{-9} \text{m}}{0.17} \approx 0.6 \times 10^{-9} \text{m} < 1 \text{Angstrom}$$

$\Rightarrow d$ must be approximately as small as spacing between atoms !!!

$\Rightarrow$ Use to examine crystal structure (e.g., spacing of atoms)

Bragg's Law:

$$2d \sin \theta_{\text{reflected}} = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots.$$
Linearly Polarized Light

Linearly polarized light
Unpolarized Light

Unpolarized light (traveling out of the page) is a superposition of many EM waves that have electric fields polarized in different directions. Example: Emitted from an ordinary light bulb.
Polarizing Light By Preferential Absorption

Polaroid
(a material with oriented long-chained molecules that selectively transmits light whose electric field is oriented along these molecules and absorbs light that has its E-field oriented perpendicular to these molecules.)
Crossed Polarizers

Second polarizer oriented the same as first:
→ All the polarized light passes through the Second polarizer.
Crossed Polarizers

Second polarizer oriented perpendicular to first:
→ All the polarized light is absorbed in the second Polarizer.
→ No light passes through second polarizer.
Crossed Polarizers

Second polarizer oriented at angle to first:
→ Only the component of E-field in direction of second polarizer gets through
→ Transmitted Intensity is proportional to $E_2$.

$E_2 = E \cos \theta$

$I_2 = I \cos^2 \theta$

(Malus's law)
Polarizing Light by Reflection

incident

reflected

$\mathbf{n}_1$

$\mathbf{n}_2$

refracted
Polarizing Light by Reflection

incident

Reflected light polarized

\[ \frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} ; \quad \theta_1 = \theta_p \quad \text{when} \quad \theta_p + \theta_2 + 90^\circ = 180^\circ \rightarrow \theta_p = 90^\circ - \theta_2 \]

\[ \frac{n_2}{n_1} = \frac{\sin \theta_p}{\sin(90^\circ - \theta_p)} = \frac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p \quad \Rightarrow \quad \tan \theta_p = \frac{n_2}{n_1} \quad \text{“Brewster Angle”} \]