Electric current: A net flow of charges.

Average current through A: \[ I_{\text{avg}} = \frac{\Delta Q}{\Delta t} \]

Instantaneous current through A: \[ I = \frac{dQ}{dt} \]

SI units of current: \( 1 \text{A} = 1 \frac{\text{C}}{\text{s}} \) "ampere"
Electric Current Direction

**Definition:**
The direction of the current is in the same direction as the flow of positive charge.

$\vec{I}$ The direction of current is opposite to the direction in which negative charge flows.
**Charge Carriers**

**Metals:**
Electrons are the charge carriers (negative charge flows).

**Electrolytes:**
Typically both positively and negatively charged ions contribute to the flow of charge.
Electric Field in Conducting Wires

Closed loop:
\( \Delta V = \text{constant in wire} \)
\( \Rightarrow E = 0 \text{ inside the wire} \)
\( \Rightarrow \text{Electrons do not move on average.} \)

Battery in the loop:
\( \Delta V \) exists from one end of the wire to the other end.
\( \Rightarrow E \) inside the wire
\( \Rightarrow \text{Electrons move within the wire opposite to the electric field in the wire.} \)
\( \Rightarrow \text{Electric current flows through the wire in the direction of electric field.} \)
(\text{charge carriers are NOT in an electrostatic equilibrium}).
**Assumption:** Charge carriers move with a drift speed $v_d$.

Only charge carriers that are in the gray cylinder will pass through cross sectional area $A$ in a time interval $\Delta t$.

The amount of charge in the gray cylinder is:

$$\Delta Q = Nq$$

where $N = \#$ of charge carriers in the gray volume.

$$N = n A \Delta x$$

where $n = \#$ of charge carriers per unit volume (charge carrier density).
Microscopic Model of Current in a Metal

\[ \Rightarrow \quad \Delta Q = n A \Delta x q = n A v_d \Delta t q \]

Average current in the conductor:

\[ I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = \frac{n A v_d \Delta t q}{\Delta t} = n A v_d \quad q \]
Example: Drift Speed in a Copper Wire

\[ I_{\text{avg}} = n A v_d q \]

\[ \Rightarrow v_d = \frac{I_{\text{avg}}}{n A q} \]

**Assume:** 12-gauge copper wire \((A=3.31 \times 10^{-6} \text{m}^2)\)

- \(I_{\text{avg}} = 1.0 \text{ A} \) (for running a typical light bulb)
- \(q = 1.60 \times 10^{-19} \text{C} \) (magnitude of the charge of an electron)

**Need:** \(n\) (volume density of free electrons in copper)

\[ n = \frac{\text{# of electrons}}{\text{Volume}} = \frac{\text{Avogadro's #}}{\text{Volume of 1 mole of copper}} = \frac{N_A}{\text{molar mass/density}} = \frac{\rho_{\text{copper}} N_A}{m_{\text{copper}}} \]

(there is one free electron per copper atom)

\[ n = \frac{8.92 \text{g/cm}^3 \cdot 6.02 \times 10^{23} \text{electrons}}{63.5 \text{g}} = 8.46 \times 10^{28} \text{electrons/m}^3 \]
Example: Drift Speed in a Copper Wire

12-gauge copper wire \((A=3.31\times10^{-6}\text{m}^2)\)

\(I_{\text{avg}}=1.0\ \text{A}\) (for running a typical light bulb)

\(q = 1.60\times10^{-19}\text{C}\) (magnitude of the charge of an electron)

\(n=8.46\times10^{28}\ 1/\text{m}^3\)

\[ v_d = \frac{I_{\text{avg}}}{n\ Aq} = \frac{1.0\text{ cm/s}}{8.46\times10^{28}\ \text{1/m}^3 \cdot 3.31\times10^{-6}\ \text{m}^2 \cdot 1.60\times10^{-19}\text{C}} = 2.2\times10^{-5}\ \text{m/s} \]

Assuming the current is always in the same direction, how long would it take an electron to drift through 5 m of wire?

\[ \Delta t = \frac{\Delta x}{v_d} = \frac{5.0\text{ m}}{2.2\times10^{-5}\ \text{m/s}} = 2.3\times10^5\ \text{s} \approx 63\ \text{hours} \approx 2.6\ \text{days} \]
Current Density (current per unit area)

Current Density: \[ J \equiv \frac{I}{A} = \frac{nA\nu_d}{A} = n\nu_d q \]

\[ J = n\nu_d q \]
(assuming uniform current density and \(A \perp\) direction of current)

For many materials: \( J = \sigma E \)

“conductivity of the material”

Such materials obey "Ohm's Law": \( \frac{J}{E} = \sigma = \text{const.} \Rightarrow "\text{ohmic" materials}"

"Nonohmic" materials do not obey "Ohm's Law"
Ohm’s Law Rewritten

For constant electric fields: \( \Delta V = EL \) (L = distance)

\[ J = \sigma E = \sigma \frac{\Delta V}{L} \quad \Rightarrow \quad \Delta V = \frac{L}{\sigma} J = \frac{L}{\sigma A} = \left( \frac{L}{\sigma A} \right) I = RI \]

\[ R \equiv \frac{\Delta V}{I} \]

, where \( \frac{L}{\sigma A} = R \) is called the resistance of the conductor.

SI units of resistance: \( 1 \Omega = 1 \frac{V}{A} \)

Resistivity: \( \rho = \frac{1}{\sigma} \) (the inverse of conductivity)

\[ R = \frac{L}{\sigma A} = \rho \frac{L}{A} \]
A Model for Electrical Conduction (Paul Drude’s model in 1900)

Average speed between collisions: approx. $10^6$ m/s

Drift speed approx. $10^{-4}$ m/s

\[ F = qE \implies \vec{a} = \sum \frac{\vec{F}}{m} = \frac{qE}{m_e} \]

$\vec{v}_i$ = velocity of electron immediately after collision (at $t = 0$)

Assumptions: $\vec{v}_i$ is random after each collision $\implies \vec{v}_{i,avg} = 0$

Excess energy gained by acceleration in el. field is lost in collisions to the atoms.
A Model for Electrical Conduction (Paul Drude’s model in 1900)

\[ \vec{v}_f = \vec{v}_i + \dot{\vec{a}} t = \vec{v}_i + \frac{q \vec{E}}{m_e} t \quad \text{(a short time } t \text{ after a collision)} \]

\[ \vec{v}_{f,\text{avg}} = \vec{v}_{i,\text{avg}} + \frac{q \vec{E}}{m_e} t_{\text{avg}} = \frac{q \vec{E}}{m_e} \tau = \vec{v}_d \]

where \( \tau = \text{average time between collisions.} \)

\( \tau \) depends on: Size of metal atoms and on \( n \) (# of electrons per unit volume).

\[ J = n \ q \ v_d = n \ \frac{q^2 E}{m_e} \tau = \sigma \ E \]

\[ \Rightarrow \sigma = n \frac{q^2 \tau}{m_e} \quad \text{and} \quad \rho = \frac{m_e}{n q^2 \tau} \]

Conductivity and resistivity in terms of microscopic quantities.
Superconductivity

$R=0$ for some materials below a critical temperature.

(Useful in superconducting magnets $\Rightarrow$ no permanent power supply necessary, but needs constant cooling)
New circuit element: The resistor

Potential Energy Change of Charge $+q$:

$a \rightarrow b : \quad \Delta U = q\Delta V$

Pot. energy increases for the charge, but chemical pot. energy in battery decreases.

$c \rightarrow d : \quad \Delta U = \text{negative}$

Pot. energy decreases for the charge. The potential energy is lost in collisions to the atoms in the resistor (the resistor heats up).

$b \rightarrow c \text{ and } d \rightarrow a : \quad \Delta U \approx 0$
Electrical Power

As charge passes through resistor:

\[
\frac{dU}{dt} = \frac{d}{dt} (Q\Delta V) = \frac{dQ}{dt} \Delta V = I \Delta V
\]

(loss of potential energy per unit time)

\[
\text{Energy per unit time} = \text{Power} \quad \Rightarrow \quad P = I \Delta V
\]

\[
P = I \Delta V \quad \text{and} \quad R = \frac{\Delta V}{I}
\]

\[
\Rightarrow P = I \Delta V = I^2 R = \frac{\Delta V^2}{R}
\]

Units of power:

\[
1 \text{ Watt} = 1 \frac{J}{s} = 1 \frac{AV}{s} = 1 \frac{A^2 \Omega}{s} = 1 \frac{V^2}{\Omega}
\]