**Physics 7740, Fall 2008**

**Problem Set 2.** Due Monday, September 15.

1. Show that $\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\times\nabla)\times\mathbf{B} + (\mathbf{B}\times\nabla)\times\mathbf{A} + \mathbf{A}(\nabla\cdot\mathbf{B}) + \mathbf{B}(\nabla\cdot\mathbf{A})$.

2. Exercise 2.5.20.

3. The quantum-mechanical angular momentum operator is $\mathbf{L} = -i(\mathbf{r}\times\nabla)$, and $L^2 = L_x^2 + L_y^2 + L_z^2$. Show that the Laplacian and $L^2$ in spherical polar coordinates are related by the equation

$$\nabla^2 = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - L^2\right]$$

You may use any results you can find in the book. Candidates: Eq. (2.48), the answers to 2.5.12 and 2.5.13, and the equations in 2.5.14.

4. (a) If $\epsilon_{ijk}$ is the Levi-Civita symbol in three-dimensional space, show that

$$\sum_k \epsilon_{ijk}\epsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}\,.$$

(b) In three-dimensional space (cartesian coordinates) the moment of inertia tensor $I_{ij}$ for a particle of mass $m$ at the point $(x_1, x_2, x_3)$ has the form

$$I_{ij} = m(\delta_{ij}(x_1^2 + x_2^2 + x_3^2) - x_i x_j)\,.$$

Show that, in terms of the rank-2 tensor $M_{ij} = m^{1/2}\sum_k \epsilon_{ijk}x_k$, $I_{ij}$ can be written as the contraction

$$I_{ij} = -\sum_k M_{ik}M_{kj}\,.$$

5. Prolate spheroidal coordinates $(\xi, \eta, \varphi)$ are defined with respect to two foci at points separated by a distance $R$ on the $z$ axis of a cartesian system: A at $(0, 0, -R/2)$, B at $(0, 0, +R/2)$. Letting $r_a$ and $r_b$ be the respective distances of a point P from A and B, P has coordinates $\xi = (r_a + r_b)/R$, $\eta = (r_a - r_b)/R$. The coordinate $\varphi$ is the azimuthal angle of P about the $z$ axis. This is an orthogonal coordinate system.

Find the metric coefficients $h_\xi$, $h_\eta$, and $h_\varphi$ and then construct the Laplacian operator $\nabla^2$ in prolate spheroidal coordinates.