Physics 7740, Fall 2008

Problem Set 3. Due Monday, September 22.

1. You derived an expression for the Laplacian ∇^2 in prolate spheroidal coordinates (ξ, η, φ) as an exercise in last week's problem set. Now show how the Helmholtz equation $\nabla^2\psi + k^2\psi = 0$ can be separated into one-dimensional equations in prolate spheroidal coordinates by introducing the ansatz $\psi(\xi, \eta, \varphi) = X(\xi)H(\eta)\Phi(\varphi)$. You should give the respective equations satisfied by X, H, and Φ and indicate the nature of the coupling between them. Do not attempt to solve any of the one-dimensional equations.

2. Obtain general solutions to the following differential equations:

$$\text{(a) } 2xyy' + 2y^2 - 3x = 0$$

$$\text{(b) } y' + y\cos x = \frac{1}{2}\sin 2x$$

$$\text{(c) } (1 + x^2)y' + y = \tan^{-1} x$$

3. Consider the differential equation

$$(1 - x)y'' + xy' - y = (1 - x)^2$$

Verify that one solution to the associated homogeneous equation is $y = x$. Use this fact to obtain a second solution to the homogeneous equation and, from these two solutions, a particular integral of the original inhomogenous equation. Combine these functions to exhibit the general solution to the differential equation.

4. Consider the differential equation

$$xy'' + (2 - x)y' - 2y = 0$$

(a) Obtain a solution, regular at the origin, in closed form (which if necessary you can discover by first obtaining a series solution).

(b) Then obtain a second solution, of the form

$$A(x)\ln x + B(x)\,,$$

with $B(x)$ a power series (possibly containing negative powers) which you will not be able to sum in closed form in terms of elementary functions. In your answer leave $B(x)$ as a series.