I. INTRODUCTION

This project report gives a detailed description and operation guide for an instrument which uses Faraday rotation of a linearly-polarized laser beam to directly measure the number density of Rb metal vapor in glass cells designed for producing polarized \(^3\)He and \(^{129}\)Xe gas using spin-exchange optical pumping (SEOP). The design of the instrument is based primarily on the apparatus described in Vliegen, et al. [1].

Alkali metal number density is a critical parameter in establishing the efficiency of the SEOP process, and an instrument which can provide a direct, relatively accurate measurement of this number density can be a useful diagnostic tool. The instrument described here is intended to be used to provide just such a diagnostic measurement of Rb number density in an existing SEOP lab setup.

Section II of the report begins with a brief overview of SEOP and a discussion of the spin-exchange rate equation and its dependence on alkali metal number density. It provides a brief review of the Faraday effect and discusses its applicability to the SEOP setup, and concludes with a discussion of the equation which relates Faraday rotation to Rb number density.

Section III provides a detailed description of the components of the instrument, which include an extended-cavity diode laser (ECDL), optics and electronics used to measure Faraday rotation, and a forced-air oven and magnetic coils.

Section IV shows sample Faraday rotation data gathered from several Rb–\(^3\)He and Rb–\(^{129}\)Xe SEOP cells, and compares the measured Rb number densities with the well-known Killian formula for the saturated vapor pressure [2].

Section V summarizes the results of the project, including a brief discussion of possible ways in which the instrument and the measurement process could be improved and refined in the future for use with SEOP setups.

Finally, Appendix A is intended to serve as a kind of “owner’s and operator’s manual” for the instrument. It includes a description of the ECDL components (power supply and temperature
controller), advice on aligning the extended cavity and tuning the laser output, and a discussion of the detection system and measurement procedure. Appendix B shows the error propagation calculations used for the sample data, and Appendix C contains additional instrument diagrams.

II. MOTIVATION AND THEORY

A. Hyperpolarized Gas and Spin-Exchange Optical Pumping

Scientists have developed a number of methods for producing gases of spin 1/2 atoms such as $^3$He and $^{129}$Xe with high degrees of nuclear magnetic spin polarization. In recent years these “hyperpolarized” gases have been used in conjunction with NMR techniques in a variety of applications in physics, medicine, and materials science, among others.

Spin-exchange optical pumping (SEOP) is a method of producing hyperpolarized gas which involves using a circularly polarized laser beam applied in the direction of the magnetic field to spin-polarize atoms of an alkali metal such as Rb (optical pumping), which then collide with noble-gas nuclei and transfer their polarization (spin-exchange). Fig. 1 shows an overview of the SEOP process, and the following sections provide a brief overview of optical pumping and spin-exchange, following the descriptions given in [3] and [4].

1. Optical Pumping

The goal of the optical pumping phase of SEOP is to produce and maintain a large number of spin-polarized alkali metal atoms. This is achieved in practice by using a pumping laser with a wavelength at a resonant atomic transition to transfer spin angular momentum to a vapor of alkali metal, creating a large population difference in the energy states of the alkali metal atoms which favors one spin orientation over the other.

In the case of Rb metal, the optical pumping process involves the following sequence of steps: (see Fig. 2):

1. Right-circularly polarized light with a wavelength at the Rb $D_1$ resonance line (794.7 nm) excites an Rb atom from the ground $5S_{1/2}, m_S = 1/2$ state to the excited $5P_{1/2}, m_J = -1/2$ state. Left-circular light would excite atoms from the $m_S = -1/2$ ground state to the $m_J = 1/2$ excited state. In the case where the applied external magnetic field is in the direction of the pumping beam, RCP is preferable to LCP because the resulting Rb polarization is in the low-energy configuration (magnetic moment pointed parallel to the field). This avoids the possibility of masing, i.e., in
\[ \lambda = 795 \text{ nm circ.} \]

polarized light

\[ \lambda / 4 \text{ plate} \]

oven 160–180ºC

\[ B \approx 30 \text{ G} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

\[ ^3\text{He} \]

FIG. 1: Overview of spin-exchange optical pumping. (a) Experimental apparatus for optical pumping. (b) Binary spin-exchange collision.

\[ m_J = \frac{1}{2} \quad (2) \quad m_J = -\frac{1}{2} \]

\[ ^5\text{P}_{1/2} \]

\[ ^5\text{S}_{1/2} \]

\[ m_S = \frac{1}{2} \quad (3) \quad m_S = -\frac{1}{2} \]

FIG. 2: Optical pumping of Rb with right-circular (\( \sigma^- \)) polarized light at 794.7 nm (modified from [4])
the high-energy configuration, a stimulated emission transition to the low-energy spin orientation which cascades to surrounding spins, resulting in a catastrophic loss of polarization.

(2) Collisions between excited Rb atoms and noble gas atoms rapidly and randomly reorient the spins of the excited Rb atoms, resulting in equal numbers of excited-state \( m_J = 1/2 \) and \( m_J = -1/2 \) atoms. This phase is frequently referred to as collisional mixing.

(3) Under ordinary circumstances, excited Rb atoms would be free to return to the ground state by emitting a photon. In the worst case, a photon with the opposite polarization of the pumping beam is emitted. In an optically thick Rb vapor, these photons can actively spoil Rb polarization as they are absorbed by surrounding ground-state Rb atoms. In order to prevent such so-called radiation trapping, nitrogen gas is added to the system to allow excited Rb atoms a non-radiative mechanism to return to the ground state by colliding with \( \text{N}_2 \) molecules. The collisional cross-section of this quenching gas is surprisingly large—a few percent \( \text{N}_2 \) by volume is sufficient to avoid losses associated with radiation trapping.

(4) Assuming that there is plenty of pumping light available, a Rb atom which returns to the \( m_S = 1/2 \) ground state will quickly absorb another pumping photon and re-enter the above process, keeping the \( m_S = 1/2 \) state depopulated as long as the pumping light is on. A Rb atom which returns to the \( m_S = -1/2 \) ground state, on the other hand, will become optically transparent to the pumping light and remain in the \( m_S = -1/2 \) ground state; it is sometimes described as having been “shelved,” since it is no longer interacting with the pumping light.

As this process proceeds through the pumping cell, more and more Rb atoms are pumped into the \( m_S = -1/2 \) state, allowing the pumping light to reach additional Rb atoms, quickly creating the desired high degree of polarization in the Rb vapor, again assuming that there is enough pumping light available.

2. Spin-Exchange and Rate Equations

The second part of the SEOP process is the transfer of polarization between the electron spin of the optically pumped alkali metal atoms and the nuclear spin of the noble gas atoms. For \(^3\text{He}\), the main polarization transfer mechanism is binary collisions between atoms, while for \(^{129}\text{Xe}\), the formation and breakup of van der Waals molecules is significant at lower gas pressures, with binary collisions becoming more important at higher pressures.

Spin-exchange collisions involve a number of specific interactions between alkali metal and noble gas atoms. These include “good” interactions, like the Fermi contact interaction, which
is responsible for transferring polarization to the noble gas atom, and “bad” interactions, such as the spin rotation interaction and the anisotropic hyperfine interaction, which destroy alkali metal polarization without any spin transfer to the noble-gas nucleus. The combination of these interactions determines the intrinsic efficiency of the spin-exchange process.

Besides intrinsic spin-exchange effects, there are also bulk mechanisms which tend to depolarize the noble gas over time. These include self-relaxation collisions between noble gas atoms, and collisions with the walls of the optical pumping cell or storage cell. The “leaky bucket” analogy is frequently used to describe the situation. Optical pumping and “good” spin-exchange collisions work to fill the bucket, while “bad” spin-exchange collisions, self-relaxation, and wall relaxation are leaks which tend to empty it.

Noble gas polarization over time can be described in more quantitative terms by means of a rate equation. In the case of $^3$He, for example, the time rate of change of noble gas polarization can be given as

$$\frac{dP_{\text{He}}}{dt} = ka[Rb](\langle P_{\text{Rb}} \rangle - P_{\text{He}}) - \Gamma_1 P_{\text{He}} + kb[Rb](P_{\text{Rb}}/2 - P_{\text{He}})$$

(1)

where $[Rb]$ is the Rb number density, $\langle P_{\text{Rb}} \rangle$ is the average Rb polarization. $\Gamma_1$ is the relaxation rate due to wall collisions and bulk self-relaxation, and is frequently referred to simply as the wall relaxation rate, since wall collisions are the dominant effect. $ka$ and $kb$ are rate constants which express the physics of their corresponding interactions. The $ka$ term represents the interaction due to Fermi contact collisions, while the $kb$ term represents the anisotropic hyperfine interaction. The Rb polarization in the $kb$ term is negative, indicating polarization in the opposite direction. The factor of 2 is a consequence of the fact that not all of the terms in the anisotropic hyperfine interaction are “bad.”

Solving Eq. 1 for $^3$He polarization as a function of time gives the exponential function

$$P_{\text{He}}(t) = \langle P_{\text{Rb}} \rangle \frac{(ka - kb/2)[Rb]}{\gamma_{se} + \Gamma_1} \left[ 1 - e^{-(\gamma_{se} + \Gamma_1)t} \right]$$

(2)

where $\gamma_{se}$ is the intrinsic spin exchange rate

$$\gamma_{se} = (ka + kb)[Rb].$$

(3)

In practice $kb$ is often assumed to be much smaller than $ka$, and drops out, leaving the simplified rate equation

$$P_{\text{He}}(t) = \langle P_{\text{Rb}} \rangle \frac{\gamma_{se}}{\gamma_{se} + \Gamma_1} \left[ 1 - e^{-(\gamma_{se} + \Gamma_1)t} \right]$$

(4)
where

\[ \gamma_{se} = k_a [\text{Rb}] \].

(5)

By examining these equations we can better illustrate some of the optical pumping and spin-exchange concepts discussed above. First of all, Eq. 3 shows that \( \gamma_{se} \) depends directly on Rb number density. Increasing the amount of Rb present in a pumping cell increases the rate of \(^3\text{He}\) polarization, as we would expect, since more Rb atoms mean more spin-exchange collisions. The caveat is that we need polarized Rb atoms. As Eq. 2 shows, \(^3\text{He}\) polarization is directly proportional to the average Rb polarization, which in turn depends on the amount of optical pumping light available. If we have too much Rb or not enough light, the Rb polarization will suffer and \(^3\text{He}\) polarization will be limited. Efficient SEOP therefore depends critically on both the Rb number density and the available power of the pumping laser.

The rate equation also allows us to see the effects of depolarizing mechanisms on the steady-state maximum polarization. In the limit as \( t \) goes to infinity, the exponential term cancels out and we are left with

\[
\lim_{t \to \infty} P_{\text{He}} = \langle P_{\text{Rb}} \rangle \frac{(k_a - k_b/2)[\text{Rb}]}{\gamma_{se} + \Gamma_1}.
\]

(6)

Assuming that all of the values are expressed as positive numbers,

\[
(k_a - k_b/2)[\text{Rb}] < (k_a + k_b)[\text{Rb}] + \Gamma_1
\]

and the \(^3\text{He}\) polarization will always be less than the ideal one-to-one ratio with Rb polarization, as expected. The maximum obtainable polarization depends on the wall relaxation rate \( \Gamma_1 \), and the “bad” spin-exchange coefficient \( k_b \).

B. Motivation for Alkali Metal Vapor Density Measurement

Because alkali metal vapor plays such a central role in the spin-exchange optical pumping process, an accurate means of determining the vapor density is important and useful both in the direct study of spin-exchange optical pumping, and in its practical laboratory applications in producing hyperpolarized gas.

Accurate measurement of alkali metal vapor density is very important in experiments designed to measure the spin-exchange rate coefficients and investigate the effects of wall relaxation on the spin-exchange process. For example, in the case of \(^3\text{He}\) and Rb, the total spin-exchange rate in the
exponential term of the above rate equation

\[ \Gamma_{\text{He}} = (k_a + k_b)[\text{Rb}] + \Gamma_1 \]  

(7)

is linear in Rb density. This relationship has been used to determine the rate coefficient \( k_a + k_b \) (or \( k_a \), assuming \( k_b \ll k_a \)) by measuring the polarization rate \( \Gamma_{\text{He}} \), or spin-up rate, as a function of measured Rb density. The wall relaxation rate \( \Gamma_1 \) is determined by measuring the rate of \(^3\text{He}\) depolarization after the pumping laser is turned off and the cell is allowed to return to room temperature and low Rb density, often referred to as a “cold” spin-down. Note particularly that this method makes the assumption that the wall relaxation rate is independent of temperature, which may not necessarily be the case.

Chann et al. [3] have developed an additional method of measuring the rate coefficients which rearranges the steady-state maximum polarization given by Eq. 6 to give

\[ k_a - k_b/2 = \frac{P_{\text{He}} \Gamma_{\text{He}}}{(P_{\text{Rb}})[\text{Rb}]} \]  

(8)

By independently measuring each of the four terms on the right, their rate-balance method can be used to measure \( k_a - k_b/2 \). By combining this method with the above spin-up rate and wall relaxation rate measurement methods, they were able to demonstrate that either \( k_b \) is larger than previously thought, or else the wall relaxation rate shows temperature dependence similar in character to that of the Rb vapor density. Their method depends on direct measurement of both Rb density and polarization, and they use Faraday rotation measurement techniques similar to those described in this report for both.

Aside from investigation of the properties of SEOP, vapor density measurement can also be useful as a diagnostic check on the efficiency of a given pumping apparatus. For example, the high-power diode laser bars currently in use as a source of pumping light have emission bandwidths on the order of nanometers, and require frequency narrowing and tuning using an extended cavity to direct their power into the desired 794.7 nm Rb atomic transition. By measuring the Rb vapor density in the pumping cell and using Eq. 6 with the determined values for \( \gamma_{se} \) and \( \Gamma_1 \), we can find the expected maximum \(^3\text{He}\) polarization the apparatus should be capable of producing. A directly measured maximum \(^3\text{He}\) polarization which differs appreciably from the expected value would suggest lower-than-expected Rb polarization, which could be due to problems with extended cavity feedback/alignment, insufficient illumination of the pumping cell, or even an unrecognized physical phenomenon.

Direct measurement of vapor density in this case is not strictly necessary—it is also possible to estimate the Rb vapor density in the pumping cell by using Killian’s empirically-determined
saturated vapor pressure formula [2], for Rb

$$\log_{10} N = 26.41 - \frac{4132}{T} - \log_{10} T$$

(9)

where \( N \) is number density in \( \text{cm}^{-3} \) and \( T \) is temperature in Kelvin. By measuring the temperature around the pumping cell, the vapor density can be calculated. Unfortunately, the true Rb vapor density in a pumping cell can vary from the value predicted by Killian’s formula by as much as a factor of 2 or more. This is especially true with a high-powered pumping laser, which heats the gas in the cell and raises the vapor pressure significantly more than the ambient temperature and Killian’s formula would indicate [3].

Measurement of alkali metal vapor density should also prove useful in the new technique of hybrid SEOP [5], where both Rb and K vapors are present in a pumping cell in order to combine the advantages of the high-powered lasers available for Rb optical pumping with the high spin-exchange efficiency of K. A small amount of Rb vapor is optically pumped, and collides with a larger amount (approximately 10:1 K to Rb ratio) of K vapor, which is quickly polarized by alkali-alkali spin exchange. The polarized K vapor participates in spin-exchange collisions with the noble gas atoms, but with a higher spin-exchange efficiency than Rb, meaning that for the same amount of light, many more K atoms can be in the vapor. The practical upshot of this approach is that we can greatly increase the spin-exchange rate without sacrificing alkali metal polarization. The actual vapor pressures of K and Rb in this case are described by Raoult’s Law of partial vapor pressures, making it much less practical to estimate them with external temperature measurements, especially since the macroscopic ratio of K and Rb metal introduced into a pumping cell can be difficult to estimate in the first place. Direct vapor pressure measurements of both K and Rb, on the other hand, would make it easier to measure and investigate the optimal ratio of K to Rb for hybrid SEOP.

Having discussed the importance of alkali metal vapor density measurement in SEOP, we now turn to a particularly suitable method of measuring it: Faraday rotation.

C. Faraday Rotation

Michael Faraday first observed the effect which bears his name in 1845, observing that a linearly polarized ray of light passing through glass (and other substances) parallel to the lines of force of a magnetic field had its direction of polarization rotated in the same direction as the current in the coil producing the field [6]. Faraday found that the rotation was proportional to the applied
field and the length of the material, and found that reflecting the ray back through the material continued to rotate the polarization in the same direction, something which is not true of substances which display simple optical activity.

The empirical formula which describes Faraday rotation of a linearly polarized beam through a particular material is

$$\theta = \mathcal{V}Bl$$

where the rotation angle $\theta$ is in arc minutes, the magnetic field $B$ is in Gauss, and the length is in cm. The constant of proportionality $\mathcal{V}$ is known as the Verdet constant; tables of Verdet constants have been compiled for various materials. For a given material, the Verdet constant also shows a dependence on the frequency of light, where higher frequencies generally imply a larger constant and more rotation.

A simple conceptual model of Faraday rotation can be constructed by imagining a linearly polarized beam of light driving an elastically bound electron into a dipole oscillation. When an external magnetic field is applied in the direction of the light (i.e., perpendicular to the oscillation) the oscillating electron feels a Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. Application of the right-hand rule shows that the force will be in one direction while the electron is oscillating up, and the opposite direction while the electron is oscillating down, causing the plane of oscillation and hence the polarization direction of the re-emitted light to be rotated slightly. In bulk material, the net result is a rotation of the plane of polarization of light as it is absorbed and re-emitted by many such oscillators.

More detailed models of Faraday rotation, either classical or quantum mechanical, rely on the important concept that a linearly polarized beam of light can be treated as a superposition of equal amounts of left-hand circularly polarized (LCP) and right-hand circularly polarized (RCP) light. An external magnetic field parallel to the direction of propagation results in a slightly different index of refraction for each helicity of light as it travels through a material, with the difference depending on the strength of the field. Similar to the case of a wave plate or other retarder, this difference in the indices of refraction results in a difference in optical path length between LCP and RCP light, and a corresponding phase delay. The accumulated phase delay while passing through the material results in an observed rotation of the direction of linear polarization

$$\theta = \frac{2\pi}{\lambda} \left( \frac{n_{\sigma^+} - n_{\sigma^-}}{2} \right) l$$

where $n_{\sigma^+}$ and $n_{\sigma^-}$ are the indices of refraction for each circular polarization, and $l$ is the length of material traversed.
In the more detailed classical description [7], the difference in index of refraction between LCP and RCP light is caused by a Lorentz force acting on an elastically bound electron, similar to the simple model above. In this case, the electron is driven into a circular path by the rotating electric field vector of the incident circularly polarized light, with the direction of rotation dependent on whether LCP or RCP light is incident. When an external magnetic field perpendicular to the plane of the electron’s path is applied, the electron feels a Lorentz force directed radially either towards or away from the center of the circle, depending on the direction of the rotation and the direction of the magnetic field. This causes a change in the radius of the circular path, and consequently in the electric dipole moment, the permittivity, and the index of refraction, resulting in one set of values for LCP light and another set of values for RCP light, and producing a phase delay and Faraday rotation.

In the quantum mechanical description, the concept of superposition of LCP and RCP light is essential, since the spin angular momentum of a photon is either $+\hbar$ or $-\hbar$, corresponding to LCP and RCP, respectively: the notation $\sigma^+$ and $\sigma^-$ is frequently used. The different indices of refraction for $\sigma^+$ and $\sigma^-$ light are described as a result of the Zeeman splitting in the atomic energy levels caused by the external magnetic field. Although the full quantum mechanical treatment is well beyond the scope of this project report, Ref. [8] provides a simple energy level model and description of Faraday rotation in the vicinity of atomic transitions which demonstrates its properties in terms of dispersion, i.e., frequency-dependence of the index of refraction, and is particularly applicable to understanding Faraday rotation in a vapor of Rb atoms.

The model considers a gas of atoms of number density $N$. Fig. 3 shows a model energy diagram for an individual atom, with a nondegenerate ground state and an excited state which shows “normal” Zeeman energy level splitting in the presence of a magnetic field $\mathbf{B} = B\hat{z}$, where the energy shifts for the $m \pm 1$ states are $\Delta E = \pm \mu_B B$. The quantization of photon spin angular momentum imposes the selection rule $\Delta m = \pm 1$, meaning that $\sigma^+$ and $\sigma^-$ photons will each interact with the model atom in a separate two-level system, with the associated transition frequencies shifted by $\pm \mu_B B/\hbar$.

The macroscopic variation in the index of refraction of the model gas near an atomic resonance depends on the microscopic dispersion properties of the two-level system. In the model of Ref. [8], the function for dispersion as a function of frequency is given by

$$n(v) - 1 = \frac{N\lambda^3}{32\pi^3\tau} \frac{v_0 - v}{(v - v_0)^2 + (\Delta v/2)^2}$$

where $v_0$ is the transition frequency, $\tau$ is the spontaneous decay lifetime, and $\Delta v$ is the natural
FIG. 3: Energy diagram showing Zeeman energy level splitting of the excited state and allowed transitions of $\sigma^+$ and $\sigma^-$ light. (Modified from Ref. [8].)

linewidth; note also that the dispersion is proportional to the number density $N$. When a magnetic field is applied, the resulting Zeeman splitting will produce two different transition frequencies $v_0$ for $\sigma^+$ and $\sigma^-$ light, and there will be two different dispersion curves given by Eq. 12. This situation is shown in Fig. 4, which shows the offset dispersion curves as a function of detuning from the original unshifted atomic transition.

Fig. 4 shows that at frequencies away from resonance, i.e., on the left-hand and right-hand sides of the plot, the splitting of the dispersion curves creates a difference in the indices of refraction, and consequently, a Faraday rotation, which drops off with increasing detuning from resonance. In the vicinity of the anomalous dispersion regions ($dn/d\nu$ negative) near resonance, there is a difference in the indices of refraction of the opposite sign, creating a “resonant” Faraday rotation in the opposite direction.

For Rb vapor, the atomic transitions and hyperfine splitting are more complex than this simple model, and the resulting the Faraday rotation signal across an Rb atomic resonance shows quite a bit of structure; examples can be seen in Refs. [8] and [9]. In the “normal” Faraday rotation regime away from resonance, however, the rotation signal is not affected by the hyperfine splitting (at least in the first-order approximation), making analysis much simpler [9]. In addition, the high Rb number density required by SEOP ($\sim 10^{14}$ cm$^{-3}$ at a temperature of 160–180 °C) results in an optically thick vapor with significant absorption near the atomic resonance. For these reasons, the Faraday rotation which is used to measure Rb vapor density is the “normal” Faraday rotation, well away from the atomic resonance.

Returning to the simple model, the strength of the applied magnetic field determines the amount
FIG. 4: Plot showing dispersion as a function of frequency detuning for $\sigma^-$ and $\sigma^+$ circularly polarized light near the atomic resonance in the Zeeman splitting model of Fig. 3. $\nu = 0$ corresponds to the unshifted absorption peak. (Modified from Ref. [8].)

of energy level splitting, and hence the offset between the $\sigma^+$ and $\sigma^-$ dispersion curves and the resulting Faraday rotation. For normal Faraday rotation, a larger magnetic field implies a larger Faraday rotation, as can be seen in Fig. 4. By applying a known magnetic field, then measuring the amount of rotation observed at a particular detuning from resonance, we can use an expression similar to Eq. 12 to determine the corresponding value of the number density $N$.

Rigorous application of the quantum mechanical model of Faraday rotation to alkali metal vapor [9] can be used to produce such an expression which relates Faraday rotation, magnetic field, and number density for Rb. The expression used in this project report is given by [3]:

$$\theta = \frac{[\text{Rb}]}{18mhc} \frac{e^2 \mu_B B}{\Delta_{1/2}^2 + 7 \Delta_{3/2}^2 - 2 \Delta_{3/2}^2 \Delta_{1/2}^2},$$  \hspace{1cm} (13)

where $\theta$ is the rotation angle in radians, $[\text{Rb}]$ is the Rb number density in cm$^{-3}$, $l$ is the length in cm, $B$ is the magnetic field in Gauss, $\Delta_{1/2}$ and $\Delta_{3/2}$ are the frequency detunings from the D$_1$ ($5P_{1/2} \rightarrow 5S_{1/2}$) and D$_2$ ($5P_{3/2} \rightarrow 5S_{1/2}$) resonances, and $e$, $\mu_B$, $m$, $h$, and $c$ are, respectively, the electron charge, Bohr magneton, electron mass, Planck constant, and speed of light in cgs units. This expression is valid for detunings well away from the absorption line.

Figure 5 shows a plot of Eq. 13 as a function of wavelength for Rb. In the vicinity of each
FIG. 5: Plot of Eq. 13 showing Faraday rotation as a function of wavelength in the vicinity of the Rb $D_1$ and $D_2$ resonances. Absorption line, the Faraday rotation decreases with increasing detuning, as we would expect from normal Faraday rotation in the simple model discussed above. Note that the infinities at the absorption lines are not physical; the expression is only valid at detunings much greater than the absorption linewidth and hyperfine structure. In reality, the more complex structure discussed above would be observable across the absorption lines.

Eq. 13 provides the necessary link which allows us to make Faraday rotation measurements of Rb number density in SEOP lab setups. We have to know the strength of the applied magnetic field and the amount of Rb vapor we are traversing, and we have to be able to measure Faraday rotation at a specific frequency detuning from an Rb atomic transition. In the next section we discuss the tunable diode laser, detection optics, and other equipment which allows us to do this.

III. INSTRUMENT DESCRIPTION

This section provides a detailed description of the various components of the Faraday rotation instrument, which include the following:

- Tunable Extended-Cavity Diode Laser (ECDL)
- Forced-air Oven and Magnetic Coils (existing equipment)
• Polarization Detection Optics and Electronics

Fig. 6 shows a high-level block diagram of the instrument.

![Diagram of apparatus block diagram](modified from [1])

**A. Tunable Extended-Cavity Diode Laser (ECDL)**

In order to be usable for SEOP Faraday rotation measurements, a laser system needs to meet the following requirements:

- **Tunability:** The laser must be frequency tunable to the vicinities of the necessary Rb and/or K absorption lines.

- **Polarization:** The output beam of the laser system should be linearly polarized. Depending on the detection system used, the linearly polarized beam is either used as-is or converted to circularly polarized light with a quarter-wave plate. For this instrument, linearly polarized light is required.

- **Linewidth:** The laser linewidth must be narrow enough to allow detuning from the alkali metal absorption line on the order of 0.5–2.0 nm without significant absorption of the laser light.

- **Power:** The laser must have sufficient output power to provide sufficient light to the detection system through optically thick alkali metal vapors 5–10 cm in length.
- Stability: The mode and output intensity noise characteristics of the laser must be stable enough to allow polarization rotation measurement of the beam.

Previously, only expensive and difficult to maintain Ti-Sapphire lasers met the necessary criteria for laser work near the Rb and K atomic transitions. However, the popularity of consumer electronics such as compact-disc burners which use near-IR laser diodes mean that there are now a wide variety of low-cost, high-quality single-mode laser diodes available in the 770-800 nm range with output power ranging from 5 to 80 mW or more. These devices, combined with frequency-narrowing extended or external cavity optics, low-noise power supplies, and appropriate temperature stabilization, can produce linearly-polarized beams with extremely narrow linewidths and tunability on the order of 10 nm around their free-running wavelength, and are very suitable for Faraday rotation measurements.

The laser used for this Faraday rotation instrument is an extended-cavity diode laser (ECDL) in the Littrow configuration, with power supplied by a commercial diode laser power supply, and temperature stabilization provided by a thermoelectric cooler (TEC), thermistor, and a temperature controller capable of proportional, integral, derivative (PID) control. Fig. 7 shows the high-level block diagram.

FIG. 7: Block diagram of laser system showing power supply and temperature controller connections

The Littrow configuration creates an extended laser cavity by placing a diffraction grating in front of the collimated light from the laser diode and orienting it so that the first-order light off the grating is reflected directly back through the laser diode. This configuration is sometimes referred to as autocollimation mode: the laser diode facet, collimation optics, and grating are arranged so that the facet is at the focal point and the first-order light from the grating is imaged directly back onto it. The critical autocollimation angle $\theta_c$ of the grating with respect to the input beam can be
obtained by using the familiar grating equation

$$\sin I + \sin D = \frac{m\lambda}{a}$$

(14)

and setting $I = D = \theta_c$. Substituting and solving for $\theta_c$, we get

$$\theta_c = \arcsin\left(\frac{m\lambda}{2a}\right)$$

(15)

where $\lambda$ is wavelength and $a$ is the groove spacing. The zeroth-order reflection off of the diffraction grating becomes the output beam of the combined ECDL system. Fig. 8 shows a diagram of the Littrow cavity arrangement with $\theta_c$ at 45°.

FIG. 8: Diagram showing Littrow arrangement of laser facet, collimating optics, and diffraction grating.

The angular dispersion properties of the diffraction grating mean that the light which is imaged back into the laser diode in the Littrow configuration has a narrow frequency range. This creates frequency-selective feedback in the laser diode junction which greatly enhances the output of the laser in that frequency range. The result is a zeroth-order output beam which can have a much narrower linewidth than the laser would produce without grating feedback. In addition, the feedback frequency range can be varied by changing the grating angle, allowing the ECDL output to be tuned away from the free-running laser diode wavelength by as much as 5 nm in either direction, depending on the alignment of the system and the amount of feedback. As mentioned earlier, it is this ability to tune the output of the laser with cavity feedback that is most useful for our Faraday rotation instrument.

There are a number of additional properties besides grating angle which need to be taken into account when considering the tunability of an ECDL in practice. The emission characteristics of
the diode laser also depend on the internal cavity properties of the laser diode, the amount of current supplied, the cavity modes formed by the extended cavity, and the amount of feedback power. A combination of all of these factors determines whether the laser will emit at a given wavelength. This produces “holes” in the tuning range obtainable by adjusting the position of the grating: rather than smoothly changing wavelength as the grating angle is changed, the laser output jumps discretely from mode to mode, with both smaller and larger jumps possible. For the purposes of this Faraday rotation instrument, these tuning “holes” do not present a significant problem. The cavity lengths used in common ECDL designs produce sufficiently smooth tuning to meet the detuning and stability requirements listed above, provided that the extended cavity is well-aligned and the current supplied to the laser diode is adjusted properly.

There are a wide variety of ECDL designs available, with varying arrangements and adjustability of laser diode, collimation optics, and grating. The design chosen for this ECDL is that of Hawthorn et al. [10], which is a modification of the original design of Arnold et al. [11]. These designs are beneficial in that they are made up mainly of commonly available components, they are simple to construct when compared with other designs, and they are relatively low-cost. Figures 9 and 10 show the main components of the Arnold-Hawthorn design, and the following paragraphs describe each component in detail.

The design uses a Newport Ultima 100-P mirror mount as its central component. The mirror mount baseplate is milled into an L shape, leaving the axis adjustment actuator screws at each corner. The post adapter is drilled out to the diameter of a Thorlabs LT110A collimation tube which accepts a diode laser package (either 9mm or 5.6mm with adapter) and a C110TM (f = 6.24mm, AR-coated) aspheric collimating optic. The collimation tube is held in place inside the post adapter ring by tapping a hole in the side of the post adapter and using a nylon or nylon-tipped screw. The modified mirror mount is positioned so that the collimation tube and laser output are horizontal, with the L-shaped baseplate on the laser output side, and the laser power cable and adjustment screws on the other. The post adapter and collimation tube remain fixed, while the baseplate is adjustable along the x, y and z axes.

A custom-machined aluminum grating mount is attached to one leg of the baseplate, and the diffraction grating is fixed onto the adapter so that it sits in front of the collimation tube, forming the Littrow cavity and allowing adjustment of the grating position with the adjustment screws. The Hawthorn design [10] attaches a second mount onto the baseplate. This mount holds a small 12.7 mm AR-coated mirror, which moves with the grating when the baseplate position is adjusted. Without this mirror, a small adjustment of the grating position can produce a significant
angular displacement of the beam, making it difficult to keep the laser aligned with the detection optics when the laser tuning is adjusted. Because the mirror rotates with the grating, the angular displacement of the beam off the grating becomes only a slight linear displacement of the beam off the mirror, resulting in a fixed-direction output beam which is much easier to work with in practice.

Fig. 9 shows an overhead view of the ECDL, showing the orientation of the modified U100-P mirror mount, the collimation tube and post adapter, and the custom grating and fixed-direction output beam mirror mounts.

![Fig. 9: ECDL top view (modified from [10])]({})

The laser used for the ECDL is a Sanyo DL-7140-201S, which has an 80 mW max output and a typical lasing wavelength of 785 nm, and is very similar to the 70 mW version used in the Hawthorn design. The particular laser diode currently in use in the ECDL for this instrument has a wavelength near the minimum range for the DL-7140-201S: the measured free-running wavelength is approximately 777 nm at room temperature. With proper cavity alignment and power adjustment, it is easily tunable to the 780 nm Rb transition, and potentially tunable to the K transition near 770 nm by temperature-tuning the free-running wavelength down a few nanometers.

Laser diodes have a naturally elliptical beam profile, and emit linearly polarized light with a polarization direction perpendicular to the long axis of the beam spot ellipse. In order to cover as
many diffraction grating lines as possible, the Arnold design recommends orienting the output of
the laser diode so that the long axis of the ellipse is perpendicular to the lines of the grating, i.e.,
horizontal. This means that the polarization direction of the output beam is vertical and parallel
to the lines of the grating. Because grating reflectivity is a polarization-dependent phenomenon,
the amount of first-order feedback obtained depends on the polarization orientation of the incident
beam. With the polarization parallel to the grating lines, minimum first-order feedback is obtained,
and it is desirable to use a diffraction grating with high reflectivity to obtain as much feedback as
possible.

The grating chosen for this ECDL is a 12.7×12.7 mm² Edmund R43-775 aluminum-coated
holographic grating, with 1800 lines/mm and a quoted absolute efficiency of approximately 40% at
780 nm. A direct measurement of zero- and first-order beam intensity with the blaze angle arrow
pointed in the direction of the first-order beam and the beam polarization oriented parallel to the
grating lines shows a relative efficiency of roughly 15%. The efficiency of this grating is expected
to be lower than the gold-coated gratings recommended in the design, however, the first-order
feedback obtained is still sufficient to meet the tunability and stability criteria for this Faraday
rotation instrument. A gold-coated grating might improve the performance of the ECDL, however.

Applying Eq. 15 to this grating with 1800 lines/mm and \( \lambda = 780 \text{nm} \), we get a critical Littrow
angle \( \theta_c = 44.6^\circ \), which is conveniently close to 45°. The faceplates of both the grating mount
and the beam-correcting mirror mount are milled to a 45° angle, and the adjustment screws are
turned to obtain the exact Littrow angle.

As described above, alignment of the Littrow cavity and output tuning require precise position-
ing of the diffraction grating with respect to the diode laser, as well as careful adjustment of the
collimating lens in order to image the selected portion of the first-order grating feedback onto the
diode laser facet as exactly as possible. The Arnold and Hawthorn designs both use piezoelectric
elements under the grating and/or between the adjustment screw and the front plate (see Fig. 9)
in order to provide fine grating adjustment and wavelength scanning. For the purposes of this
Faraday rotation instrument, however, the rough frequency tuning provided by the adjustment
screws proves to be sufficient, since the detuning from the atomic transition is not required to be
any particular wavelength, and wavelength scanning is unnecessary.

The collimating optic for this laser was carefully adjusted so that the laser spot size remained
the same over a distance of 6-10 feet. The Arnold design suggests that in order to optimize cavity
alignment and frequency narrowing, an adjustment tube should be glued onto the end of the
collimating optic, and the collimation should be further adjusted to maximize the output power
with the feedback condition in place. For this instrument, this adjustment tube was omitted, and the collimation and frequency narrowing may therefore not be optimal. The feedback and tunability obtained meets the requirements for measuring Faraday rotation, but applications such as spectroscopy which require optimal linewidth narrowing would almost certainly require more careful collimation adjustment.

Power for the laser is supplied by a Melles-Griot 06 DLD 201 low-noise diode laser driver, attached to the laser with a Thorlabs SR9C cable. The SR9C cable mates directly with the laser diode package, provides ESD protection circuitry, and has a strain relief housing designed to screw directly onto the LT110A collimation tube. Details and pinouts for the cable can be found in Appendix C.

Temperature stabilization for the laser is provided by a feedback system which uses a 100 KΩ thermistor for temperature sensing, and a 40 W Peltier thermoelectric cooler (TEC) to cool the laser assembly. The thermistor is mounted into a hole drilled on the side of the modified post adapter so that it is as close to the laser as possible. The entire ECDL assembly is mounted onto a flat rectangular mounting plate and attached to a large aluminum base block, which provides both mechanical stability and a large thermal mass to act as a heatsink for the TEC. The TEC is sandwiched between the mounting plate and the base block, with the cold side towards the mounting plate, and the hot side towards the base block. All surfaces are coated with heatsink compound, and anodized surfaces are sanded away wherever possible in order to provide good thermal contact between the various parts of the system. Fig. 10 shows a side view of the laser assembly showing the arrangement of the mounting plate, TEC, and base block.

![FIG. 10: ECDL side view](image-url)
The thermistor forms one leg of a Wheatstone bridge, where the other resistances are chosen to provide approximately linear output over a temperature range of 10–30 °C. The Wheatstone bridge circuit is connected to a commercial Omega temperature controller unit, which provides proportional, integral, derivative (PID) feedback control output to a Darlington transistor and power supply, which are used to power the TEC. The desired setpoint temperature is set on the front panel of the temperature controller unit, which adjusts the control output to achieve and maintain the setpoint temperature on the thermistor input. The Omega temperature controller, power supply, and associated circuitry are enclosed in an aluminum project box, and banana plug jacks and cables are used to connect the thermistor and TEC power to the laser assembly.

With proper grating alignment and operation near room temperature, the ECDL for this project was found to be tunable between approximately 773 and 781 nm. The lasing threshold occurs at approximately 30 mA input current, and the laser is typically operated at 40–50+ mA input current, depending on the sensitivity of the photodetectors, light absorption by the Rb vapor, and the feedback and stability requirements of the extended cavity. In order to provide temperature stabilization, the temperature controller setpoint is set a few degrees below ambient room temperature, typically 17-19 °C. The cavity alignment and laser wavelength is monitored by passing the beam through a piece of plate glass and observing the weak reflected beam with an Ocean Optics spectrometer.

The linewidth of the laser has been measured with a Fabry-Perot interferometer with a cavity length of 15 cm and an associated free spectral range of approximately 1 GHz. The observed laser line through the Fabry-Perot in this configuration had an estimated FWHM of 250 MHz, however, there were problems with the Fabry-Perot ramp generator which prevented further measurements, and the cavity alignment when the measurement was taken was almost certainly sub-optimal. Although the ECDL linewidth appears to be more than narrow enough to be useful for Faraday rotation measurements, it would be a good idea to make a more accurate measurement of the narrowed linewidth.

Some trial and error is necessary to find a combination of laser current and cavity alignment which has stable frequency and power output characteristics over long periods of time. Adjustment of laser current is particularly prone to causing undesirable mode hops in the laser output, which also seem to change the output polarization characteristics of the laser enough to affect polarization measurements. In addition to these larger mode hops, which can be largely eliminated by careful current adjustments, what appears to be a very small, slow oscillation in the laser polarization state has also been observed, which causes some difficulty in making accurate Faraday rotation
measurements for very small rotation angles. This oscillation may be the result of smaller mode hops, or some other type of laser noise, and might be caused by the power supply and/or the extended cavity characteristics. It may be desirable to build a cover for the ECDL assembly to rule out the possibility that air currents could be affecting the cavity stability. Improvements listed above, such as a higher reflectivity grating, a piezoelectric element for finer grating position adjustment, and more careful collimation adjustment may also help.

Additional details of the operation of the ECDL, including procedures for aligning the extended cavity, can be found in Appendix A.

B. Detection Optics and Electronics

The polarization measurement apparatus used in this instrument is modeled after the one used by Vliegen, et al. [1]. It has relatively modest requirements in terms of optics, and is simple to set up and use. Fig. 11 shows a block diagram of the detector system.

![Block diagram of Faraday rotation system showing the arrangement of the focusing lens, rotatable half-wave plate, beamsplitter cube, photodetectors, and the connections to the difference amplifier and oscilloscope](image)

The heart of the polarization measurement system is a 15 mm Edmund R47-047 polarizing beamsplitter cube used to separate the linearly-polarized probe beam into orthogonally polarized components. It has a 780 nm antireflective coating, a transmission efficiency of 95% for incident P-polarized light, and a reflection efficiency of 99.5% for incident S-polarized light. It is attached to a mirror mount with tip-tilt adjustment in order to facilitate alignment with the photodetectors and other optics.
A matched pair of photodetectors is used to measure the intensity of the orthogonally-polarized beams emitted from the beamsplitter cube. Each photodetector consists of a 3.2 mm$^2$ silicon photodiode (Edmund H53-372) with a 9V reverse bias applied and a 10 KΩ potentiometer connected to ground to provide voltage output with adjustable sensitivity (see Appendix C for a circuit diagram). The photodetector circuitry is contained inside small aluminum project cases, and a piece of IR filter glass is placed over the window of each photodiode to prevent stray room light from interfering with measurement. The photodetectors are matched by adjusting the sensitivity of each to produce approximately the same voltage output for a given laser input. In order to focus the probe laser light into the photodetectors, a 25 cm plano-convex lens with an AR coating is placed in in front of the beam as shown in Fig. 11.

The other critical component of the polarization measurement system is a CVI QWP0-795-10-2 zero order half-wave plate, mounted on a ThorLabs PRM1 precision rotation mount with a micrometer screw capable of up to 1 arc minute resolution. The retardance properties of the half-wave plate are used to orient the linear polarization of the probe beam with respect to the axes of the beamsplitter cube. By rotating the half-wave plate, the polarization of the probe beam can be rotated to any arbitrary angle and the intensity of the resulting components can be measured by the photodetectors. While this half-wave plate is designed for operation at 795 nm, because it is zero order the error in half-wave retardance between the fast and slow axes is negligibly small when used at 780 nm.

The output of each of the photodetectors is connected to a difference amplifier circuit, in this case a classic differential instrumentation amplifier built from 3 low-noise op-amps and 1% tolerance resistors, arranged to to produce a gain of 50 (see Appendix C for a circuit diagram). An oscilloscope is used to observe the output from the difference amplifier; it can also be used to look directly at the voltage output of the photodetectors, which is particularly helpful when aligning the laser and optics. The amplifier used in this instrument was built on a prototype board: this and the 1% resistors limit its performance and common mode noise characteristics, although they are sufficient to demonstrate the Faraday rotation measurement technique. A carefully assembled amplifier with higher-tolerance resistors or a commercial amplifier would be expected to produce better results.

In order to measure Faraday rotation, a null-difference method is used. The detection apparatus is arranged so that the difference signal out of the amplifier is zero with the magnetic field off. The ECDL polarization is oriented parallel to the diffraction grating lines and vertically as described above, and the half-wave plate fast axis is positioned at 22.5° from vertical, so that the polarization
rotates by 45° as it passes through the half-wave plate, resulting in equal intensities at each photodetector and a null signal out of the amplifier. Note that in practice the small difference in transmission and reflection efficiency of the beamsplitter cube means that the sensitivity of one of the photodetectors must be adjusted slightly to achieve this condition and keep the component measurement properties consistent.

When the magnetic field is turned on, Faraday rotation produced by the Rb in the cell changes the angle of the polarization with respect to the fast axis of the half-wave plate by an amount $\theta$; after passing through the half-wave plate the polarization direction incident on the beamsplitter cube is changed by the same angle $\theta$. This causes a change in the component intensities measured by the photodetectors, producing a nonzero difference signal out of the amplifier. The half-wave plate is then rotated by an angle $\phi$ in order to restore the polarization direction incident on the beamsplitter cube and re-null the difference signal. The angle $\phi$ by which the half-wave plate was rotated is then used to determine the actual Faraday rotation angle: $\theta = 2\phi$, since the half-wave plate rotates linear polarization by twice the angle with the fast axis. When the magnetic field is turned off, the polarization returns to its original position, and a difference signal of the same magnitude is observed from the amplifier. Rotating the half-wave plate back to its original position once again re-nulls the difference signal.

In practice, the rotation angles produced at the temperatures and magnetic fields applicable to spin-exchange optical pumping are very small: around 30 arc minutes of half-wave plate rotation for an oven temperature of 190 °C and a magnetic field of 40+ Gauss, corresponding to a Faraday rotation of only a degree or so. The resolution of the rotating stage and the amplifier gain are very important to making accurate rotation angle measurements, as is identification and control of noise sources. Appendix A and Section IV contain additional details of the measurement process and techniques for reducing the impact of noise.

C. Oven and Magnetic Coils

The oven used in this project is the same type of forced-air oven used for spin-exchange optical pumping, capable of heating a cell up to more than 200°C. An existing oven was used for this experiment. Since the oven construction is outside the scope of this project, only a brief description will be given. The oven consists of an insulated box with circular glass windows on 3 sides which allow the beam to pass through and the cell to be observed. An insulated pipe connected to the bottom of the oven supplies hot air to the interior, and an RTD is used to monitor the temperature.
The supply of hot air is provided by connecting a pressure regulator and resistive heater to the building air supply, and using an Omega temperature controller of the type used for the laser temperature controller to provide feedback-regulated temperature control using the RTD. For this project the oven was used with 15 psi of air pressure, a flow rate of approximately 150-200 SCFH, and a temperature range of 150–190 °C.

The magnetic field is provided by a pair of large Helmholtz coils placed around the oven, with power supplied by an 8.6 A power supply. A Hall-effect probe was used to measure the magnetic field in the center of the oven at maximum current: at 8.6 A the measured field was 47.1 G, and this relationship was used to convert coil current settings to magnetic fields for the example data in Section IV.

In order to reduce the effect of the Earth’s magnetic field on the cell, a second, smaller set of Helmholtz coils is placed around the oven, oriented so that the direction of the magnetic field is opposite to the direction of the Earth’s field. The Hall-effect probe was placed at the center of the oven, and current to the correcting coils was adjusted so that the probe showed zero field with the main coils off.

**IV. DATA AND RESULTS**

In order to demonstrate the operation of the instrument, Faraday rotation measurements were made on a set of 3 similar cylindrical optical pumping cells with varying gas compositions and pressures. Table I shows the cell names and properties associated with each cell.

<table>
<thead>
<tr>
<th>Cell Name</th>
<th>Contents</th>
<th>Gas Pressure (mbar)</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120A</td>
<td>$^3\text{He}+$ N$_2$, Rb</td>
<td>399</td>
<td>9.6</td>
</tr>
<tr>
<td>120B</td>
<td>$^3\text{He}+$ N$_2$, Rb</td>
<td>750</td>
<td>9.45</td>
</tr>
<tr>
<td>120C</td>
<td>$^{129}\text{Xe}+$ N$_2$, Rb</td>
<td>722</td>
<td>9.5</td>
</tr>
</tbody>
</table>

**TABLE I: Cells used for example Faraday rotation data**

In order to see Rb vapor density measurement at different temperatures, Faraday rotation data were collected on each cell at 5 oven temperatures: 150, 160, 170, 180, and 190 °C. In each case, the ECDL was detuned from the Rb $D_2$ resonance line by approximately 0.5 nm, and the laser current was set between 44 and 54 mA. See the captions of the data figures below for the precise conditions for each measurement.

At each temperature, a series of individual Faraday rotation measurements were made at 8
different magnetic field strengths, listed in Table II.

<table>
<thead>
<tr>
<th>Coil Current (A)</th>
<th>1.6</th>
<th>2.6</th>
<th>3.6</th>
<th>4.6</th>
<th>5.6</th>
<th>6.6</th>
<th>7.6</th>
<th>8.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mag. Field (G)</td>
<td>8.8</td>
<td>14.2</td>
<td>19.7</td>
<td>25.2</td>
<td>30.7</td>
<td>36.1</td>
<td>41.6</td>
<td>47.1</td>
</tr>
</tbody>
</table>

TABLE II: Coil currents and corresponding magnetic fields for example Faraday rotation measurements

For cell 120B, only a single Faraday rotation measurement was taken at each magnetic field value, using the following process:

1. Null difference signal

2. Turn on magnetic field, observe difference signal

3. Rotate half-wave plate to re-null difference signal, record resulting rotation angle

4. Turn off magnetic field, re-null difference signal

The amount of rotation was estimated to the nearest 1.2 arc minute (i.e., 1/50 of a degree) using the micrometer screw markings on the rotating stage, which are spaced at 2.4 arc minute (1/25 of a degree) increments. The measurements in the graphs below are shown in milliradians so that they can be easily substituted into the Faraday rotation expression.

For cells 120A and 120C, multiple Faraday rotation measurements were taken at each magnetic field value in order to deal with noise and periodic variations in the difference signal (discussed in additional detail below). A total of 6 measurements were made for each magnetic field value, and then averaged together to produce a mean value and associated error. In order to save time, 2 rotation measurements were made each time the magnetic field was applied, using the following procedure:

1. Null difference signal

2. Turn on magnetic field, observe difference signal

3. Rotate half-wave plate to re-null difference signal, record rotation angle (1st measurement)

4. Turn off magnetic field, observe difference signal

5. Rotate half-wave plate to re-null difference signal, record rotation angle (2nd measurement)
The amount of rotation for these cells was estimated to the nearest 0.6 arc minute (1/100 of a degree).

Random fluctuations in laser intensity were observed on the photodetectors when the ECDL beam was directed through the oven at higher temperatures, even without a cell present. They appear to be caused by small, rapidly changing temperature gradients in the hot, turbulent air of the oven. The signal averaging feature of the oscilloscope was used to reduce the impact of these fluctuations on the difference signal output.

Besides the rapid fluctuations, slower oscillations in the difference signal were also observed. It is believed that these oscillations are caused by slight changes in the beam polarization direction of the ECDL itself, perhaps due to air currents affecting the laser cavity or noise from the laser power supply, although further examination is necessary to confirm this. In any case, these oscillations can cause the difference signal to drift around the null point over a few minutes time by up to 10% or more of the full Faraday rotation signal, which causes problems, especially with measurement at low fields and very small rotation angles.

For each cell, the measured Faraday rotation angles and their errors were plotted as a function of magnetic field, and a linear fit was applied to determine the slope $\theta/B$ and its associated error. Figures 12 through 26 show the resulting data for each cell and temperature range. In the table on each figure, $m_1$ represents the y-intercept of the linear fit, and $m_2$ represents the slope.

There are a number of interesting observations to be made from this data. All of the graphs show a reasonably linear relationship between measured rotation angle and magnetic field, allowing for measurement error. As expected, the measured slopes increase with temperature, indicating increased amounts of Rb vapor. Effects of drift in the difference signal on the rotation measurements can be observed, particularly in the measurements taken on cell 120B, where only a single rotation measurement was made at each magnetic field value.

The cells containing $^3$He (120A and 120B) all show a y-intercept much greater than zero, an effect which appears to increase with temperature. Although it is not shown in the graphs provided in this project report, for each cell the measured rotation angle was observed to jump rapidly upward when the magnetic field is increased from zero. Rough measurements of the field at lower values shows a dual-slope behavior, with a very steep slope below about 10 G, transitioning to a more gradual slope as the field is increased. The second slope is believed to be the result of Faraday rotation, and calculations performed with it agree fairly well when compared to the expected Rb vapor density in each cell. Interestingly enough, cell 120C, which contains $^{129}$Xe, does not show nearly the same degree of dual-slope behavior, with y-intercepts close to zero, but still
FIG. 12: Cell 120A, 150 °C; 50.1 mA laser current, $\lambda = 779.41$ nm

FIG. 13: Cell 120A, 160 °C; 50.2 mA laser current, $\lambda = 779.46$ nm
FIG. 14: Cell 120A, 170 °C; 50.2 mA laser current, $\lambda = 779.46$ nm

FIG. 15: Cell 120A, 180 °C; 50.2 mA laser current, $\lambda = 779.46$ nm
FIG. 16: Cell 120A, 190 °C; 50.2 mA laser current, $\lambda = 779.46$ nm

FIG. 17: Cell 120B, 150 °C; 47.4 mA laser current, $\lambda = 779.46$ nm
FIG. 18: Cell 120B, 160 °C; 47.4 mA laser current, $\lambda = 779.46$ nm

FIG. 19: Cell 120B, 170 °C; 47.4 mA laser current, $\lambda = 779.46$ nm
FIG. 20: Cell 120B, 180 °C; 53.3 mA laser current, $\lambda = 779.46$ nm

FIG. 21: Cell 120B, 190 °C; 53.3 mA laser current, $\lambda = 779.46$ nm
FIG. 22: Cell 120C, 150 °C; 44.0 mA laser current, $\lambda = 779.46$ nm

FIG. 23: Cell 120C, 160 °C; 44.0 mA laser current, $\lambda = 779.46$ nm
FIG. 24: Cell 120C, 170 °C; 50.8 mA laser current, $\lambda = 779.46$ nm

FIG. 25: Cell 120C, 180 °C; 50.8 mA laser current, $\lambda = 779.46$ nm
increasing slightly with temperature.

The additional rotation observed in the $^3$He cells may be the result of some type of transverse optical pumping, where magnetic fields orthogonal to the direction of the beam produce pumping similar to the process used in optical pumping with circularly polarized light as described above. The secondary set of correcting coils described above was added to the system in order to reduce the effect of the Earth’s field, which is oriented perpendicularly to the laser and main magnetic coil direction. The additional rotation is observed even with the correcting coils: all of the data presented here was taken with the correcting coils on. Further investigation is needed to determine the cause of this additional rotation.

In order to calculate the vapor density at each temperature, the measured slope $\theta/B$, the length $l$, and the detuning from the Rb $D_2$ and $D_1$ lines $\Delta_{3/2}$ and $\Delta_{1/2}$ are inserted into the expression for Rb Faraday rotation discussed in Section II C (Eq. 13), which is rearranged to give Rb vapor density as a function of the other parameters:

$$[\text{Rb}] = \frac{\theta}{Bl} \frac{18mhc}{e^2\mu_B} \left(\frac{\Delta_{1/2}^2\Delta_{3/2}^2}{4\Delta_{3/2}^2 + 7\Delta_{1/2}^2 - 2\Delta_{1/2}\Delta_{3/2}}\right)$$ (16)

The error in the calculated Rb vapor density is obtained by propagating the error in the linear fit parameter for the Faraday rotation slope $\theta/B$, along with an estimated error $\delta l = 0.05 \text{ cm}$ for the
cell length, and an estimated detuning error $\delta \lambda = 0.01$ nm. Details of the error calculation can be found in Appendix B.

Figure 27 shows the results of the number density calculations for each cell, plotted as a function of temperature and compared with Killian’s saturated vapor pressure curve obtained with Eq. 9.

![Graph showing measured Rb number density versus temperature for cells 120A, 120B, and 120C compared with Killian’s saturated vapor pressure curve.]

FIG. 27: Faraday rotation number density versus temperature for cells 120A, 120B, and 120C compared with Killian’s saturated vapor pressure curve.

The measured Rb number density in each case was found to be within a factor of 2 of the value predicted by Killian’s empirical formula, in agreement with the understood accuracy of that formula [3]. The fact that the measured number densities are consistently lower than Killian’s formula may be due to a systematic error, or it may be that the formula tends to overestimate the actual vapor pressure present in a cell across this range of temperatures. The estimated error in the calculated Rb vapor density was found to be 10% or less, which is a significant improvement over Killian’s formula.
V. CONCLUSION

Direct, accurate determination of alkali number density is vital for research into the physics and applications of noble gases polarized by spin-exchange optical pumping (SEOP), allowing better characterization of spin-exchange rates and alkali metal polarization, and aiding in the determination of alkali number density ratios for new “hybrid” SEOP techniques using K and Rb.

This project report has described an instrument suitable for the direct measurement of Rb number density in a wide variety of spin-exchange optical pumping cells, showing example number density measurements over a range of temperatures for cells containing both $^3$He and $^{129}$Xe. The number density dependence was found to be qualitatively similar to Killian’s empirical formula for Rb number density, but actual measurements at a given temperature were found to vary from cell to cell, suggesting the need to make measurements on a cell-by-cell and experiment-by-experiment basis in order to obtain an accurate picture of the number density.

It is hoped that this instrument will be the starting point for a much more capable Faraday rotation measurement system of the type described in [3], which can measure both Rb number density and Rb polarization while optical pumping is going on. Retrofitting the existing instrument in this way would require a number of additional considerations, including:

Modification of optics to work alongside a pumping laser Currently, the ECDL output is directed into the cell using a simple periscope arrangement; this would need to be updated so that both the probe beam and the pumping beam could be directed into the cell parallel to the magnetic field. On the detector side, the optics would need to be modified to allow the probe beam to pass through to the detector while the pumping beam was blocked.

Additional equipment The direct Faraday rotation measurement approach used in this instrument is suitable for measuring large rotation angles, but becomes problematic when measuring small rotation angles. With a photoelastic modulator (PEM) and lock-in amplifier as in [3], small rotation angles could be measured more quickly and more reliably.

New experimental techniques In order to measure Rb polarization, an RF coil is added to the experimental arrangement: application of an RF field at the Rb resonance frequency results in a polarization spectrum which can be directly observed as peaks in the Faraday rotation signal as the external magnetic field is varied. In addition to polarization measurement, the Rb number density can be determined even while optical pumping is going on by using an expression similar to Eq. 13 which uses both Faraday rotation angle and Rb polarization.
APPENDIX A: OPERATIONS MANUAL

This section is intended to provide a practical guide to the use the Faraday rotation instrument described in this paper, including the operation of the ECDL, alignment and usage of the detection optics, and measurement procedure. Hopefully it will be helpful as a starting point in understanding how to adapt the equipment for use in future experiments.

1. Using the ECDL

Operation of the ECDL is basically straightforward, especially for those who are already familiar with the operation of optical pumping lasers and other diode lasers. The most difficult procedure is probably the initial alignment of the extended cavity to produce feedback-enhanced output; once this is complete the laser can be easily tuned across its full wavelength range by making small adjustments to the precision screws on the laser assembly and monitoring the output with a fast spectrometer.

Longer-term stabilization of the laser output requires careful adjustment of the power supply current, the temperature controller, and the extended cavity itself. The basic operation of the power supply and the temperature controller is briefly described below, followed by recommended methods for extended cavity alignment and tuning.

a. Operation of the Melles-Griot 06DLD201 Low-Noise Diode Laser Driver

The power supply used with this laser system is an off-the-shelf Melles-Griot laser power supply with very low noise output and some nice features which would probably not normally be found on a custom-built power supply. It has a variety of operating modes and options, only some of which will be described here. See the operation manual which accompanies the power supply for additional details.

The power supply has the following controls on the front panel (see Fig. 28):

Main Power Switch Keyswitch used to turn the power supply on and off. Power to the laser is not applied when the power is turned on initially; the output switch must be pushed to apply power to the laser.

Laser Output Adjustment Knob Used to adjust the laser output to the desired current/power level.
Display Depending on the display and mode switches, shows the max. current limit in mA (LIMIT), output current in mA (LASER), or power output of the laser (PHOTO).

Display Switch Switches the display between LIMIT, LASER, and PHOTO (only accessible when mode is set to PWR).

Mode Switch Switches the laser between constant current output mode (CUR) and constant power output (PWR) mode. The constant current output mode is used to power the 80 mW laser diode in the ECDL.

Output Switch Enable (LASER) or disable (PRESET) output to the laser; defaults to PRESET.

The back panel of the power supply (see Fig. 29) contains the following:

Line Power AC line power in.

Power Cable to Laser Diode 8-pin barrel connector used to supply current to the laser diode package. See Appendix C for a detailed pinout.
Laser Type Switch  Used to select the laser diode type. The Sanyo DL-7140-201S is Melles-Griot TYPE D, however different manufacturers have different type codes: refer to connection diagrams like the ones in Appendix C and the product datasheets when in doubt.

IMPORTANT: Do not change this setting unless you are changing the laser diode!

To operate the power supply, turn the main power switch to ON. The laser output mode will default to PRESET. Set the display switch so that it shows the laser current (LASER), make sure that the mode switch is set to constant current mode (CUR), then use the output adjustment knob to adjust the current to the desired level. In order to turn on actual current to the laser, push the output switch to switch from PRESET to LASER mode. Further adjustment to the laser output power can be made with the adjustment knob.

According to the specifications sheet, the Sanyo DL-7140-201S laser diode has a maximum operating current of 140 mA. In order to avoid accidentally exceeding this value, the power supply maximum output current limit is set to 130 mA. However, since the ECDL directs a significant amount of optical power back through the laser diode, it is recommended that the actual maximum current used in practice be kept under 80 mA or so.

The threshold current for the DL-7140-201S is around 30 mA, and typical operating current for making Faraday rotation measurements is in the vicinity of 40–60 mA, depending on the amount
of absorption through the Rb cell and the sensitivity of the detectors. As mentioned above, the current adjustment to the laser is important in tuning and stabilizing the ECDL output, since variation in injection current has an effect on the output mode of the laser diode, even at currents away from the lasing threshold [12]. Once the grating position of the extended cavity has been used to tune the laser output to the desired frequency, it is still usually necessary to adjust the output current to avoid stability problems associated with mode hopping.

b. Operation of the Temperature Controller Unit

As discussed in the main body of the paper, the temperature controller system for the ECDL consists of a thermistor (US Sensor DC104R25 100kΩ), 40 W Peltier thermoelectric cooler (TEC), and a custom-built control box which houses a DC power supply (Power-One MAP40-3000), an off-the-shelf Omega temperature controller unit (iSeries i16D dual display), and additional control circuitry. It follows the design for pumping laser temperature controllers used in our labs, and it is nearly identical to those controllers, although the power connectors on the rear of the unit are different, and there are other changes, including DC cooling fans rather than AC, an LED indicator instead of a lamp, a different fuse arrangement, and different thermistors and power supplies. This section will describe the operation of the temperature controller unit for normal day-to-day use with the laser. For additional details, including circuit diagrams and pinouts, see Appendix C. For detailed information on configuring and operating the Omega temperature controller unit, see the accompanying manual.

![FIG. 30: Front view of temperature controller box](image)

The temperature controller has the following controls and displays on the front panel (see Fig.
Main Power Switch  This switch controls the temperature controller’s DC power supply, which supplies current to the TEC, the cooling fans at the rear of the temperature controller, and the power LED. Note that the main power switch does not control the Omega temperature controller unit, which uses line AC power directly and powers on as soon as the AC power in is plugged in.

Main Power LED  The main power LED is on when power to the DC power supply is on, and off otherwise.

Upper Display (Current Temp.) During normal operation, the upper display on the Omega temperature controller unit indicates the current temperature measured at the thermistor in degrees Celsius. While using the Menu, the upper display is used to select and change configuration settings and setpoint values.

Lower Display (Setpoint Temp.) During normal operation, the lower display shows the setpoint temperature in degrees Celsius.

Menu Button  The Menu button is used to enter the Omega controller unit’s menu list, which includes the setpoint menus (SP1 and SP2) and the configuration menu, among others.

Up Button  The Up button is used to increment the currently displayed numerical value (e.g., a setpoint menu value, or the selected (flashing) portion of a selected configuration menu numerical value).

Down Button  If a setpoint menu item is selected, the down button decrements the setpoint value. If a configuration menu numerical value is displayed, the down button is used to scroll from left to right between the digits of the value. The down button is also used to return to a top-level menu.

Enter Button  The enter button is used to enter submenus from the top-level menu. After modifying a menu value, the enter button is used to store the new value (the upper display flashes “StRd” briefly). During normal operation, pressing the enter button twice in a row places the controller in standby mode (the upper display flashes “Stby” repeatedly).

The rear panel of the temperature controller has the following inputs and outputs (see Fig. 31):
**AC Power In** The AC power in connector connects the temperature controller to wall power.

**Thermistor Input** The thermistor input (yellow/green) banana plug jacks connect to the corresponding jacks on the laser assembly, and connect the thermistor mounted in the laser assembly to the bridge circuit and Omega temperature controller process input.

**TEC Output** The TEC output (red/black) banana plug jacks connect to the corresponding jacks on the laser assembly, and connect the DC power supply to the TEC mounted in the laser assembly.

![FIG. 31: Back view of temperature controller box](image)

To operate the temperature controller, turn on the main power switch. Adjust the setpoint temperature (SP1) to the desired value using the following steps:

1. Press the menu button until SP1 is displayed.
2. Press the enter button to display the current setpoint value.
3. Press the up and down buttons to adjust the setpoint to the desired value. Holding the button increases the rate at which the value changes.
4. Press the enter button to store the new setpoint value. The controller immediately begins referencing the new setpoint value, even while in menu mode.
5. Press the menu button to cycle through the remaining top-level menus and reenter normal operation mode. The current temperature should be moving toward the new setpoint value.
If it is necessary to make other configuration changes to the Omega temperature controller (e.g., performing an auto-tune PID operation after changes to the laser assembly), please see the Omega iSeries User’s Guide for detailed instructions.

The temperature controller is calibrated to show a temperature reading of 20 °C at the corresponding resistance value of the 100 kΩ thermistor, and the bridge circuit which converts the thermistor resistance to a voltage value for the Omega controller’s process input (see Section III A and Appendix C) is linearized around this value to provide approximately correct temperature readings between 10–30 °C. Since it usually isn’t necessary to adjust the laser temperature to a particular value, the loss in accuracy as the temperature moves away from the center value does not cause problems: precision and stability of the measured temperature value are more important for laser temperature control.

Unlike the pumping lasers usually used with this temperature controller design, the laser diode in this instrument is not a significant heat source. Indeed, one of the purposes of the pumping laser temperature controller is to prevent the laser from overheating, while the diode laser in this instrument can safely be operated at room temperature. One consequence of this, however, is that the design is only able to cool the laser diode assembly, requiring the setpoint temperature to be several degrees or so below the ambient room temperature, and affecting the tunability range of the laser diode. In the existing lab setup, setpoint temperatures of 17.50–18.00 °C were used, and this was found to be well within the tunability range necessary to make vapor density measurements near the 780 nm Rb D₂ resonance. Since lasing wavelength decreases with temperature (see DL-7140-201S spec. sheet), the existing temperature controller should also be able to make density measurements of K at the 770 nm K D₁ resonance with additional cooling, although this has not been tested with the existing setup.

Once the setpoint temperature is reached, the actual temperature is generally observed to be stable to within ±0.01 °C, although there are occasional variations of up to ±0.05 °C. These larger variations may be caused by air currents in the lab, and it would probably be desirable to build a cover for the laser assembly to reduce this effect, increasing the stability of laser diode temperature and hopefully improving the stability of the laser output.

c. Operation of the ECDL: Cavity Alignment and Tuning

As discussed in Section III, the extended cavity diode laser requires careful alignment of the diffraction grating and collimation optics with respect to the laser diode (see Fig. 32) to obtain
and maintain its linewidth narrowing and frequency tuning properties. This section will describe procedures for aligning the cavity and subsequent frequency tuning.

![Front view of laser assembly](image1)

**FIG. 32:** Front view of laser assembly

The position of the diffraction grating is adjusted using the 3 actuator screws of the modified mirror mount (see Fig. 33).

![Back view of laser assembly showing adjustment screws](image2)

**FIG. 33:** Back view of laser assembly showing adjustment screws

**Horizontal Axis Adj.** Rotates the position of the grating with respect to a horizontal axis. This screw is used during initial cavity alignment, and also in frequency output tuning to keep the extended cavity aligned while rotating the grating about the vertical axis.
Forward/Back Translation Adj. With corresponding adjustment of the other two screws, allows forward-back adjustment of the grating position, lengthening or shortening the extended cavity. It should not usually be necessary to adjust the position of this screw during normal laser operation.

Vertical Axis Adj. Rotates the position of the grating with respect to a vertical axis. This screw is used during initial cavity alignment, and is the primary mechanism for tuning the laser frequency output.

Aligning the extended cavity involves the following steps (see Ref. [11]). As when working with any other laser, be extremely careful to avoid accidentally directing the beam or any specular reflections into anyone’s eyes. It is a good idea to keep one’s head out of the horizontal and vertical planes of the beam as much as possible and use an IR viewer and/or an IR card to observe the path of the beam and its reflections.

1. Remove the fixed-direction output mirror from the laser. Make sure that the diffraction grating is properly positioned in front of the collimation tube (see Fig. 32). Note that the mirror and grating are attached to their mounts with rubber cement only (in order to make it easy to switch components), so care should be taken when attaching and removing them to avoid damage.

2. Turn on the laser and adjust the output to a reasonable level. Verify that the collimation tube is positioned so that the long axis of the output spot is horizontal, i.e., perpendicular to the grating lines; the collimation tube position can be adjusted by loosening the nylon screw and rotating the tube inside the collar.

3. Check that the output beam is collimated over a distance of at least a few meters by directing the beam across the room and checking the beam spot size. It may be easier to do this with an IR viewer or IR card, rather than relying on the visible appearance of the beam spot. If collimation adjustment is necessary, a spanner wrench can be used to adjust the position of the collimation optic in the tube. Note that while the provided spanner wrench has been modified so that it can be inserted into the front of the collimation tube while the grating is still attached to the laser, it is recommended that the diffraction grating be removed to avoid damaging it. (It may be desirable to purchase another spanner wrench of the correct size from ThorLabs.) As is, the beam should be already reasonably well collimated, so adjustment should not be necessary.
4. Place a piece of paper a short distance away from the laser (the periscope in the current lab setup makes a nice paper stand) and direct the beam onto it. Using an IR viewer to observe the beam spot on the paper, adjust the horizontal and vertical axis screws (and the forward/back screw, if necessary) until a second, much weaker beam spot appears in the vicinity of the main spot. This second spot is the first-order reflection off the diffraction grating, which reflects off the back facet of the laser diode, then off the diffraction grating again, making a round trip through the cavity.

5. Using the horizontal and vertical axis screws, move the first-order reflection spot so that it is very close to the zeroth-order spot; either directly to one side or above/below it. Turn the laser power output so that the laser is just below the lasing threshold. The zeroth-order spot will be faintly visible through the IR viewer, and the first-order spot will probably not be visible at all.

6. Now, use the adjustment screws to track the first-order spot directly onto the zeroth-order spot. When the first-order beam is precisely aligned, the output spot will suddenly grow much brighter as the first-order feedback brings the laser back above threshold. Continue making careful adjustments to maximize the brightness of the output spot.

7. Turn off the laser output. Reattach the fixed-direction output mirror and make sure that it is aligned properly with the diffraction grating (see Fig. 32). Turn the laser output back on. Reattaching the mirror mount may be enough to bump the cavity out of alignment, so it may be necessary to repeat the previous adjustment again to reestablish feedback.

Once the extended cavity is aligned and providing feedback near the lasing threshold, the output current can be increased as necessary. At this point, it is useful to switch from the IR viewer to a spectrometer for observing both the laser cavity alignment and output wavelength. For this project, the laser output beam was directed through a piece of plate glass, and the reflection off the plate glass was directed into a fast PC-controlled spectrometer capable of showing the laser line in real-time. With the cavity aligned, turning the vertical axis adjustment rotates the grating and changes the output wavelength, which can be directly observed with the spectrometer. Tuning the laser is simply a matter of rotating the vertical axis adjustment until the laser is at the desired wavelength.

If the cavity moves out of alignment while the grating is being rotated, the laser output will jump back to its free-running wavelength, and adjustment of the horizontal axis adjustment may
be necessary. Adjustment of the laser current can also change the output wavelength, and finding a balance between the grating position and the laser current for a desired wavelength setting may require some trial-and-error adjustment of each.

With temperature control, good cavity alignment and output current settings, and the spectrometer to monitor the output wavelength, the laser is ready to be used for Faraday rotation measurements.

2. Using the Detection Optics

Since Section III B describes the detection optics for measuring polarization rotation in some detail, this section mainly serves to provide a few additional details and tips for alignment and usage, along with photos of the experimental setup.

![Side view of detection optics](image)

**FIG. 34:** Side view of detection optics

Fig. 34 shows the arrangement of the detection optics with the beam out of the cell coming from the left side through the lens, through the half-wave plate, and polarizing beamsplitter cube, and finally to each of the photodetectors. A brief description of each component follows.

**Focusing Lens** 25 cm focal length lens used to focus the probe beam into the photodetectors.

**Rotating Stage and λ/2 Plate** Fig. 35 shows the important components of the rotating stage, including the main bezel, which is used for large-scale rotation, the micrometer screw, which
is used for fine rotation adjustment, and the locking screw, which locks the main bezel into place to allow the micrometer screw to be used. The fast axis of the half-wave plate is aligned (as closely as possible) with the 0° mark on the bezel.

![Diagram of rotating stage components](image)

**FIG. 35:** Top view of the rotating stage

**Polarizing Beamsplitter** The polarizing beamsplitter cube is oriented on a mirror mount so that horizontal polarization passes through to the right-hand photodetector in Fig. 34, while vertical polarization is reflected to the left-hand photodetector. The tip-tilt adjustments on the mirror mount are important for aligning the beam onto the photodetectors.

**Photodetectors** The photodetectors are placed an equal distance away from the beamsplitter cube, so that the beam path for each polarization state is the same. Proper adjustment of the location of each photodetector is important for beam alignment. The photodetectors are matched by removing the top cover and adjusting the potentiometer inside so that the photodetector voltage outputs are the same for a given laser input. The alkali vapor cell will attenuate the beam intensity appreciably near resonance, so the sensitivity of the photodetectors should be adjusted to use the full voltage range, but be careful not to direct too much intensity into the photodetectors. In typical usage, the photodetector will be getting around half of the total beam intensity through the beamsplitter cube.

Fig. 39 shows an overhead view of the full apparatus. Alignment of the beam through the most of the components is basically straightforward. It is important to make sure that the beam is as horizontally level as possible: this makes getting the beam through the oven, cell, and into the photodetectors easier. In the experimental apparatus shown, this is done by using the tip-tilt adjustment on the periscope (visible in front of the ECDL on the left-hand side) and measuring the height of the beam at various points on the optical breadboard which holds the detection
FIG. 36: Overhead view of the experimental apparatus showing alignment of components

optics. To align the photodetectors and beamsplitter cube and prepare the apparatus for taking measurements, the following procedure can be used:

1. Turn on the laser and align it through all optics, including the beamsplitter cube. Use the main bezel on the rotating stage to rotate the half-waveplate to 45°. The beam out of the extended-cavity diode laser is vertically linearly polarized (long axis of the beam spot is horizontal), so this rotates the beam 90° and allows maximum transmission through the beamsplitter cube.

2. Turn on the first photodetector (right-hand side on Fig. 34 and connect its voltage output to one channel of an oscilloscope. Adjust the position of the photodetector and the beamsplitter tip-tilt to maximize the measured voltage. Also adjust the rotation of the half-waveplate to find the maximum transmission, since the fast axis might not be at exactly 0°.

3. Rotate the main bezel on the rotating stage to 0°, which allows maximum reflection off of the beamsplitter cube. Turn on the second photodetector (left-hand side on Fig. 34 and connect its voltage output to the other channel of the oscilloscope. Once again, adjust the position of the photodetector, beamsplitter tip-tilt, and half-waveplate rotation to maximize the measured voltage, but make sure that you are not throwing off the alignment of the first photodetector.

4. With the oscilloscope showing both photodetector outputs, rotate the half-waveplate through
a 45° range, and verify that the intensity goes from one photodetector to the other smoothly and linearly; make additional position and tip-tilt adjustments as necessary. The maximum voltage at each photodetector should be roughly the same, but because the beamsplitter cube transmission efficiency is about 4% lower than the reflection efficiency, it may be desirable to make a slight adjustment to the sensitivity of one of the photodetectors to match the maximum observed voltage (make sure that any difference in maximum voltage really is due to transmission efficiency, and not misalignment).

5. Rotate the half-waveplate so that the voltage output at each photodetector is equal (null difference); this should be very close to 22.5° on the rotating stage, and corresponds to a 45° incident polarization on the beamsplitter cube.

With the initial alignment completed, the oven can be turned on and brought up to temperature. While the oven is heating, the signal measured by the photodetectors oscillates up and down, and noise due to turbulent air in the ovens becomes pronounced. As the alkali metal vapor density increases, the beam intensity measured at the photodetectors also decreases when the laser is tuned near resonance.

Once the oven has reached its setpoint temperature and is stable, readjust the laser current as necessary, and rotate the half-waveplate to reestablish the null difference. If there is a pronounced amount of noise in the photodetector output, it will be necessary to carefully adjust the tip-tilt on the beamsplitter cube and the photodetector position to reduce the noise. A certain amount of noise is unavoidable (it can be dealt for measurements by using oscilloscope signal averaging), but a considerable amount of noise can be caused by the beam tracking off the edge of the photodiode surface, especially if only one of the photodetectors is affected.

The photodetector outputs can now be switched to the difference amplifier, and the amplifier output can be connected to a single oscilloscope channel. Null the difference signal again by gently bumping the main bezel of the rotating stage into position; the micrometer screw can also be used. Without signal averaging, the air current noise should be very pronounced through the difference amplifier. It may be necessary to further adjust the beamsplitter cube and photodetector position. When the laser switches modes, it also has an effect on the polarization, and this can cause sudden jumps of the difference signal that make measurement difficult. It is usually possible to find a stable region of laser output where the null signal remains constant, but it requires some trial-and-error adjustment of the laser current, cavity alignment and tuning, and half-waveplate rotation.

At this point, the detection system is ready to be used for rotation measurements. The Faraday
rotation signal can be measured by applying the magnetic field, noting the difference signal, and then rotating the half-waveplate with the micrometer screw to re-null the signal; the amount of Faraday rotation is twice the amount of half-waveplate rotation, as discussed in Section III B. One turn of the micrometer screw corresponds to one degree of rotation, and there are 25 divisions marked on the screw, so one division corresponds to 1/25 of a degree. The following procedure can be used to quickly take 2 rotation measurements at a given magnetic field value:

1. Null the difference signal, and rotate the micrometer screw to the 0 position, turning the locking screw so that the micrometer screw is engaged.

2. Apply the magnetic field, then quickly re-null the signal by rotating the micrometer screw. Read the rotation value off of the micrometer screw.

3. Make sure that the difference is still null, and turn off the magnetic field (re-null if the signal has wandered at all). Quickly re-null the signal again by rotating the micrometer screw in the opposite direction and read the rotation value off of the micrometer screw.

A number of these measurements can be made at each magnetic field value, and then averaged together to produce a data point.

It is also possible to read the voltage value off of the oscilloscope to determine the amount of Faraday rotation. In practice, however, the ratio of signal voltage to rotation depends on the amount of attenuation of the laser as it passes through the vapor cell, making it necessary to calibrate the voltage to the actual rotation amount for each oven temperature and assume that the calibration is good for the other data points at that temperature. A direct reading of the half-waveplate rotation, while more work, makes an absolute measurement of the rotation every time.

**APPENDIX B: ERROR PROPAGATION**

In order to estimate the error in the Rb vapor density measurements made with this instrument, we consider the expression for Faraday rotation, rearranged so that the Rb vapor density is on the left-hand side:

$$[\text{Rb}] = \frac{\theta}{B l} \frac{18 m h c}{e^2 \mu_B} \left( \frac{\Delta^2_{1/2} \Delta^2_{3/2}}{4 \Delta^2_{3/2} + 7 \Delta^2_{1/2} - 2 \Delta^2_{1/2} \Delta^2_{3/2}} \right).$$ (B1)
We then separate out the terms on the right-hand side into measured quantities and constants

\[
[Rb] = \left( \frac{\theta}{B} \right) \left( \frac{1}{7} \right) \left( \frac{18mhc}{e^2\mu_B} \right) \left( \frac{\Delta^2_{1/2}\Delta^2_{3/2}}{4\Delta^2_{3/2} + 7\Delta^2_{1/2} - 2\Delta_{1/2}\Delta_{3/2}} \right) \tag{B2}
\]

where (1) is the measured Faraday rotation slope, (2) is the measured cell length, (3) is constant, and (4) is the detuning from resonance. (1), (2), and (4), have errors associated with them, while (3) does not.

We begin with the error in (1), the Faraday rotation slope \( S = \frac{\theta}{B} \), which depends on direct measurements of rotation angle \( \theta \) and magnetic field \( B \). We plot \( \theta \) as a function of \( B \), and apply a linear fit algorithm to the resulting set of points. Assuming a random distribution of measurement errors, we can directly use the error value associated with the slope of the linear fit as our error \( \delta S \).

The error in the cell length (2) depends directly on the accuracy of our measurement. Using a ruler with millimeter markings, we assume that we can estimate the cell length to within half a millimeter, giving an estimated error \( \delta l = 0.05 \) cm.

The error in the detuning term (4) depends on the errors in the detunings \( \Delta_{1/2} \) and \( \Delta_{3/2} \). Error propagation proceeds as follows:

\[
q = \frac{\Delta^2_{1/2}\Delta^2_{3/2}}{4\Delta^2_{3/2} + 7\Delta^2_{1/2} - 2\Delta_{1/2}\Delta_{3/2}}
\]

\[
\frac{\partial q}{\partial \Delta_{1/2}} = \frac{2\Delta^2_{1/2}\Delta^2_{3/2}}{7\Delta^2_{1/2} + 4\Delta^2_{3/2} - 2\Delta_{1/2}\Delta_{3/2}} - \frac{\Delta^2_{1/2}\Delta^2_{3/2}(14\Delta_{1/2} - 2\Delta_{3/2})}{(7\Delta^2_{1/2} + 4\Delta^2_{3/2} - 2\Delta_{1/2}\Delta_{3/2})^2}
\]

\[
\frac{\partial q}{\partial \Delta_{3/2}} = \frac{2\Delta^2_{1/2}\Delta_{3/2}}{7\Delta^2_{1/2} + 4\Delta^2_{3/2} - 2\Delta_{1/2}\Delta_{3/2}} - \frac{\Delta^2_{1/2}\Delta^2_{3/2}(8\Delta_{3/2} - 2\Delta_{1/2})}{(7\Delta^2_{1/2} + 4\Delta^2_{3/2} - 2\Delta_{1/2}\Delta_{3/2})^2}
\]

\[
\delta q = \sqrt{\left( \frac{\partial q}{\partial \Delta_{1/2}} \delta \Delta_{1/2} \right)^2 + \left( \frac{\partial q}{\partial \Delta_{3/2}} \delta \Delta_{3/2} \right)^2} \tag{B3}
\]

In order to estimate the detuning errors \( \delta \Delta_{1/2} \) and \( \delta \Delta_{3/2} \), we make the relatively conservative assumption that we can measure the wavelength of the ECDL to within \( \pm 0.01 \) nm. At \( \sim 0.5 \) nm detuning from the Rb \( D_2 \) line, this implies a frequency detuning error \( \approx 5 \times 10^9 \) Hz for both \( \delta \Delta_{1/2} \) and \( \delta \Delta_{3/2} \).

Since the terms in Eq. B2 are multiplied together, we can obtain the total fractional error in the Rb vapor density \( \delta N \) by adding the fractional errors for \( \delta S \), \( \delta l \), and \( \delta q \) in quadrature:

\[
\frac{\delta N}{N} = \sqrt{\left( \frac{\delta S}{S} \right)^2 + \left( \frac{\delta l}{l} \right)^2 + \left( \frac{\delta q}{q} \right)^2} \tag{B4}
\]
FIG. 37: Thorlabs SR9C cable pinout (diode connection) and schematic. The connection type for the Sanyo DL7140-201S laser diode package is Thorlabs Style C, Melles-Griot TYPE D.

FIG. 38: Thorlabs SR9C cable pinout for Melles-Griot laser driver TYPE D connection (8-pin barrel connector; viewed from behind).
FIG. 39: Circuit diagram for matched Si photodetectors.

FIG. 40: Circuit diagram for difference amplifier circuit. The input followers are LF351N op-amps, and all resistors are 1% metal film resistors. $V_{CC} = 15$ V and $V_{EE} = -15$ V are supplied to power the op-amps, and a trimming potentiometer connected to $V_{EE}$ is also connected to one of the input followers.
FIG. 41: Circuit diagram and pinout for temperature controller process input. The thermistor is a US Sensor DC 104R25. The bridge circuit resistors are chosen to linearize the process input around 5 V for a thermistor resistance of 126.8 kΩ, which corresponds to a temperature of 20°C.

FIG. 42: Circuit diagram and pinout for temperature controller process output and AC power in. 12 V power to the TEC is provided by the temperature controller power supply, with the circuit ground left floating. The transistor is a Darlington high-gain type, and the 1 Ω resistor is rated to 30W.


