Problem 1  *Plotting Events on a Classical Space-Time Diagram*

You are walking at 2 m/s down a straight road. At a particular time you pass your friend Katrina, who is standing still. 5 s later a dog barks; at that moment he is 10 m ahead of you in the road. After another 5 s, a car backfires; at that moment it is 15 m behind you.

(a) Plot and label the events described above on a two-dimensional graph of time vs. position (space-time diagram) corresponding to your reference frame.

(b) Plot and label the same events on a space-time diagram corresponding to Katrina’s reference frame. (Assume your and Katrina’s watches are synchronized.)
Problem 2  Relative Velocity

If you throw a superball (perfectly elastic) with speed $u$ at a stationary wall, it bounces back with the same speed in the opposite direction.

(a) What happens if you throw it at speed $u$ towards a wall which is traveling towards you at speed $w$?

Figure 2: Space-time diagram in Katrina’s reference frame.

Let’s designate your reference frame as unprimed ($S$) and the wall’s frame as primed ($S'$). So the ball moves with velocity $+u$ (to the right) in the $S$ frame and the wall moves with velocity $v = -w$ (to the left) in the $S$ frame. Obviously, the relative speed between the ball and wall is $u + w$ (classically). Formally, this can be obtained by doing a classical (Galilean) velocity transformation from the $S$ to the $S'$ frame: $u' = u - v = u + w$. Thus, in the $S'$ frame, the ball approaches the wall with velocity $u' = u + w$ and will rebound with velocity $u_{reb}' = -(u + w)$.

Now transform the rebound velocity back to the unprimed $S$ frame: $u_{reb} = u_{reb}' + v = -(u + w) + (-w) = -(u + 2w)$. Thus we see that the ball rebounds

Figure 3: Superball approaching a moving wall.
with speed $u + 2w$ in your reference frame.

(b) What is the answer in the limit in which $w$ is much larger than $u$?

If $w \gg v$, then we can neglect $u$ in the above equation, and $u_{reb} = -2w$. So in this limit, the rebound velocity is independent of the ball’s initial velocity.