Relativity Problem Set 2

Prof. J. Gerton

Due Wednesday, September 7 - at the beginning of class

Problem 1  (10 pts.) Consequences of Special Relativity

(a) How fast must a clock move in order to run at one-half its proper rate?

(b) How fast must a stick move so that an observer at rest sees it shrunk to one-third its proper length?

Problem 2  (10 pts.) Relativistic car trip

The proper distance between New York and Los Angeles is 5,000 km (let’s assume this is an exact number). A car travels at 100 km/h between the two cities.

(a) What is the distance traveled by the car according to its passengers? (You may want to use a Taylor expansion.) Compare this to the 5,000 km distance measured by an observer at rest with respect to the Earth.

(b) How much time does the trip take according to the passengers? Compare this to the time it takes according to an observer at rest with respect to the Earth.

(c) Answer parts (a) and (b) for a car speed of 0.1c.

Problem 3  (10 pts.) Muon lifetime

The muon is an unstable subatomic particle that decays with a proper lifetime of 2.2 µs. At a particular time, a beam of 10^4 muons is traveling with velocity \( v = 0.9c \) with respect to the laboratory.

(a) What is the muon lifetime in the laboratory frame?
(b) How many muons survive after the beam has traveled a distance of 2 km in the laboratory frame? (Hint: the number of muons that survives after a time $t$ is given by an exponentially decaying function with a time constant given by the muon lifetime.)

**Problem 4  (10 pts.) The ladder and the barn paradox**

You want to demonstrate the Lorentz length contraction by fitting a 10 m long ladder into a 5 m deep garage. You tie the ladder on top of your car and ask your friend Bob to drive the car into the open door of the garage. You stand by the door and as soon as the rear end of the ladder passes, you shut the door proving that the ladder fits into the garage.

(a) How fast must Bob drive so that the ladder fits into the garage from your point of view?

(b) At this speed, how deep is the garage from Bob’s point of view? Does the ladder fit into the garage according to him?

(c) Draw a space-time diagram of the garage and the ladder in both your frame and Bob’s frame; use the diagram to explain the situation.

**Problem 5  (10 pts.) Time dilation and length contraction**

(a) Write an expression for the fractional change in the length of an object relative to the proper length ($\Delta L/L_0$) as a function of the speed of the object through a reference frame.

(b) Write an expression for the fractional change in the measured time interval between two events relative to the proper time ($\Delta \tau/\tau_0$) as a function of the speed of the proper reference frame. (Here $\tau_0$ is the proper time interval between the two events.)

(c) Using a graphing program (e.g., Matlab, Mathematica, Maple, etc.), plot $\Delta L/L_0$ and $\Delta \tau/\tau_0$ as a function of $\beta = v/c$ for $0 \leq \beta \leq 1$. 

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**COURSE NAME**

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