Lecture 37: Stern-Gerlach Experiment & Spin

A. Angular Momentum + Magnetic Dipole Moment

- H-atm: We found that angular momentum of the electron is quantized.
- The electron is charged $\rightarrow$ the magnetic moment of the electron's orbit is also quantized.
- In the classical picture, the current associated with the orbital motion of the electron is:

\[ I = \frac{e}{T} \]

where $e$ is the electron charge and $T$ is the orbital period.

- A current loop with current $I$ and area $A$ has a magnetic dipole moment $\mathbf{M} = I \mathbf{A} = -\frac{e}{2} \pi r^2 = -\frac{e}{2\pi r} \pi r^2$

\[ \Rightarrow \mathbf{M} = -\frac{e}{2} \mathbf{r} \mathbf{v} = -\frac{e}{2m} (m \mathbf{v} r) = -\frac{e}{2m} \mathbf{L} \]

velocity of electron

- A magnetic dipole in a magnetic field experiences a torque $\mathbf{T} = \mathbf{M} \times \mathbf{B}$, which causes the angular momentum to precess like a top:

\[ \mathbf{T} = \frac{d\mathbf{L}}{dt} \]
The precession rate is the Larmor frequency:

\[ \frac{d\vec{L}}{dt} = \frac{e}{2m_e} \hat{L} \times \vec{B} = \frac{eB}{2m_e} \hat{L}_z \sin \theta \]

\[ L \sin \theta \frac{d\phi}{dt} = \frac{eB}{2m_e} L \sin \theta \implies \frac{d\phi}{dt} = \frac{eB}{2m_e} \]

The component of along the B-field \((L_z)\) is constant while the \(x+y\) components keep changing.

The magnetic field direction sets a special axis, \(\hat{z}\), along which the angular momentum \(-\) and the magnetic moment \(-\) are quantized \(\Rightarrow L_z = m_L \hbar\)

B. Stern-Gerlach Experiment

1922 - Otto Stern and Walter Gerlach investigated the intrinsic angular momentum of atoms.

A magnetic dipole \(\vec{m}\) in a magnetic field \(\vec{B}\) has a potential energy:

\[ U = -\vec{m} \cdot \vec{B} \]

\[ \Rightarrow \text{If } \vec{B} \text{ is non-uniform, then } \vec{m} \text{ is subject to a force: } \vec{F} = -\nabla (\vec{m} \cdot \vec{B}) \] Atomic Beam along channel

For the situation at right:

\[ F = m \cdot \frac{\partial B_z}{\partial z} \hat{z} \]

\[ = \left( -\frac{e}{2m_e} L_z \right) \frac{\partial B_z}{\partial z} \hat{z} \]

\[ = \left( -\frac{e}{2m_e} \right) \frac{\partial B_z}{\partial z} \hat{z} \rightarrow m_L = l_z, \ldots, +1 \]
H-atoms in state $J=1$ should be separated into 3 spots at the other end of the magnetic channel:

- When $m_J = +1$, atoms are deflected downward
- $m_J = 0$, atoms not deflected
- $m_J = -1$, atoms deflected upward

When Stern & Gerlach performed their experiment, they used ground-state H-atoms $\Rightarrow J = 0$

$\Rightarrow$ Only one (undeflected) spot was expected.

$\Rightarrow$ They observed 2 spots, so this was something new!

C. Spin

- Electrons have an intrinsic angular momentum, $s$, which underlies an intrinsic dipole moment
- All particles are characterized by their intrinsic properties such as mass, electric charge, and spin, ...
- A proper description of spin requires relativistic quantum mechanics - Quantum Field Theory:

1) $S = \sqrt{S(S+1)} \hbar$ where $S$ is a quantum number whose value is a property of each particle.

$\Rightarrow$ Electrons have $S = \frac{1}{2}$, photons have $S = 1$

2) The intrinsic magnetic dipole moment is

$$m_s = \frac{e}{2m_e} S,$$

where $g=2$ is the "gyromagnetic ratio" of the electron, so $m_s = \frac{e}{2m_e} S$
3) \( S_z = m_S \hbar \) with \( m_S = -1, -1 + 1, \ldots, 1 \).

- \( m_S \) is the spin quantum number

\[ \Rightarrow \text{for an electron, } M_S = -\frac{1}{2} \text{ (down) or } M_S = \frac{1}{2} \text{ (up) } \]

- The spin quantum number explains the result of the Stern-Gerlach experiment:

\[ F = -\frac{e}{m_e} S_z \frac{\partial B_z}{\partial z} \hat{z} \]

\[ = -\frac{e}{m_e} m_S \hbar \frac{\partial B_z}{\partial z} \hat{z} \Rightarrow m_S = \pm \frac{1}{2} \]

- To fully specify the state of an electron in an \( H \) atom, we must add a fourth quantum number:

\( \Psi \)

\[ \{ n, l, m_l, m_s \} \]

- If the energy doesn’t depend on \( m_S \), then the degeneracy is raised from \( n^2 \) to \( 2n^2 \):

- Using “ket” notation to specify a quantum state, we can denote the “spin state” of the electron as a linear combination of up \((1\uparrow)\) and down \((1\downarrow)\) states:

\[ 1\uparrow = a_1 \uparrow + b_1 \downarrow \]

- Using “ket” notation, the spin state of an electron can be written as a superposition of up \& down spins:

\[ 1\uparrow = a_1 \uparrow + b_1 \downarrow \], where \( |a|^2 \) is the probability of measuring \((1\uparrow)\).