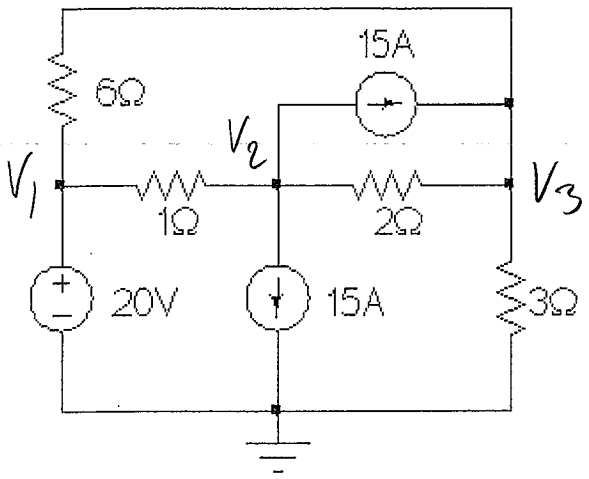
Problem 1 (13 pts):

Find the node voltages for the circuit below. (As always:) Label the circuit diagram accordingly to show what your variables mean.



$$V_1 = 20V$$

KCL @ V_2:

$$\frac{V_2 - 20V}{1\Omega} + 15A + 15A = \frac{V_3 - V_2}{2\Omega}$$

$$2V_2 - 40V + 30V + 30V = V_3 - V_2$$

$$3V_2 + 20V = V_3$$

KCL @ V_3:

$$15A = \frac{V_3}{3\Omega} + \frac{V_3 - 20V}{6\Omega} + \frac{V_3 - V_2}{2\Omega}$$

$$6\Omega 15A = 90V = 2V_3 + V_3 - 20V + 3(V_3 - V_2)$$

$$110V = 6V_3 - 3V_2$$

$$110V = 6(3V_2 + 20V) - 3V_2 = 15V_2 + 120V$$

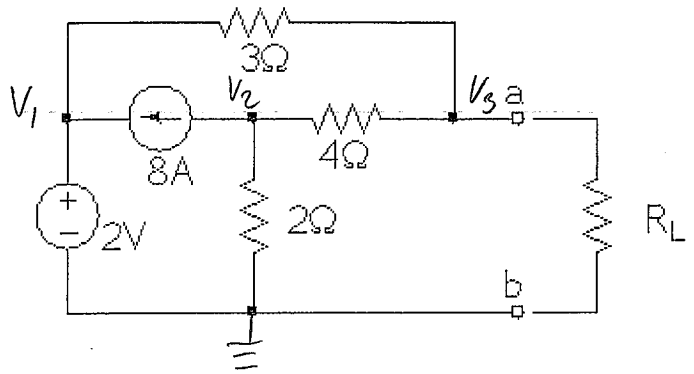
$$15V_2 = -10V$$

$$V_2 = -\frac{10}{15}V = -\frac{2}{3}V = V_2$$

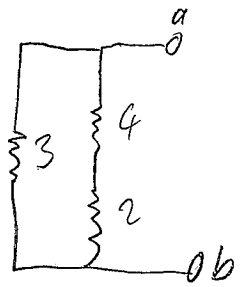
$$V_3 = 3V_2 + 20V = -2V + 20V = 18V = V_3$$

Problem 2 (10 pts):

Find the Thevenin equivalent of the circuit to the left of the terminals a and b. Use the Thevenin equivalent to find the power absorbed in the load R_L if $R_L = 3\Omega$. Draw the circuit of the Thevenin equivalent with the 3Ω load.



R_{Th}:



$$\Rightarrow R_{Th} = \frac{(4+2)3\Omega}{3+(4+2)} = \frac{18}{9}\Omega = \boxed{2\Omega = R_{Th}}$$

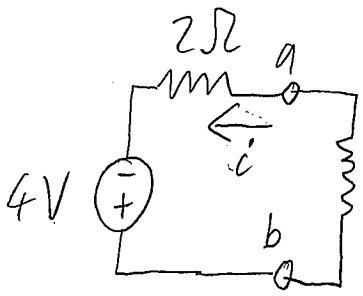
U_{Th}: KCL@V_2: $8A = \frac{V_3 - V_2}{4\Omega} - \frac{V_2}{2\Omega} \Rightarrow 32V = V_3 - V_2 - 2V_2 = V_3 - 3V_2$

$V_1 = 2V$; KCL@V_3: $\frac{V_3 - V_2}{4\Omega} = \frac{V_1 - V_3}{3\Omega} \Rightarrow 3V_3 - 3V_2 = 4V_1 - 4V_3$

$$7V_3 = 3V_2 + 8V$$

$$32V = V_3 - 3V_2 = V_3 + 8V - 7V_3 = 8V - 6V_3$$

$$U_{Th} = V_3 = \frac{24V}{-6} = \boxed{-4V = U_{Th}}$$



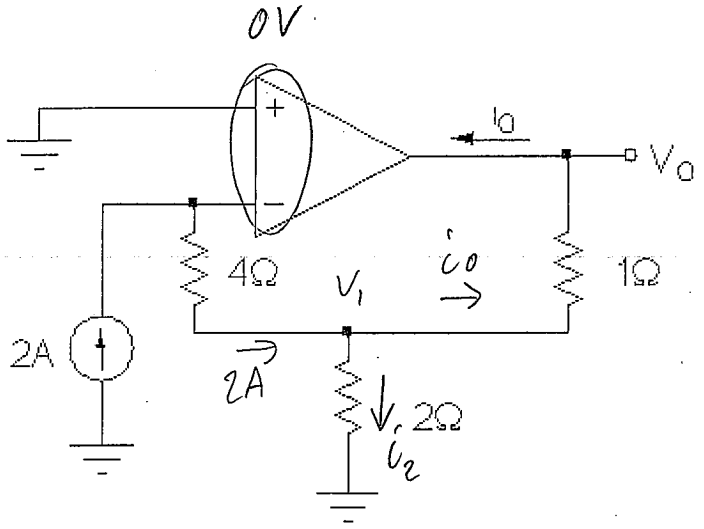
$$\Rightarrow i = \frac{4V}{(2+3)\Omega} = \frac{4}{5}A$$

$$P = Ri^2 = 3\frac{16}{25}W$$

$$\boxed{P = \frac{48}{25}W = 1\frac{23}{25}W}$$

Problem 3 (6 pts):

Find V_O and i_O:



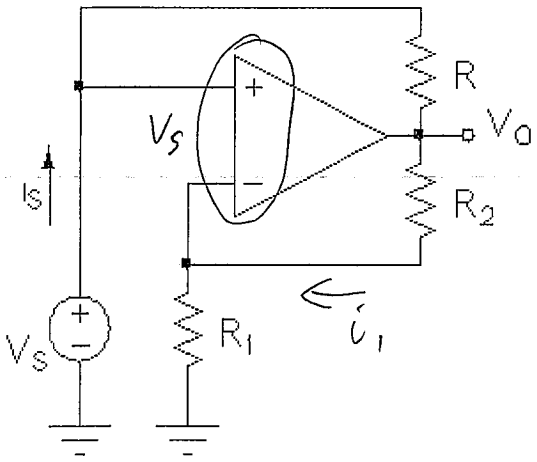
$V_1 = -4\Omega\, 2A = -8V \Rightarrow i_2 = \frac{-8V}{2\Omega} = -4A$

$i_0 = 2A - i_2 = 2A + 4A = \boxed{6A = i_0}$

$V_0 = V_1 - i_0\, 1\Omega = -8V - 6V = \boxed{-14V = V_0}$

Problem 4 (7 pts):

Find V_O and the resistance V_S/i_S:



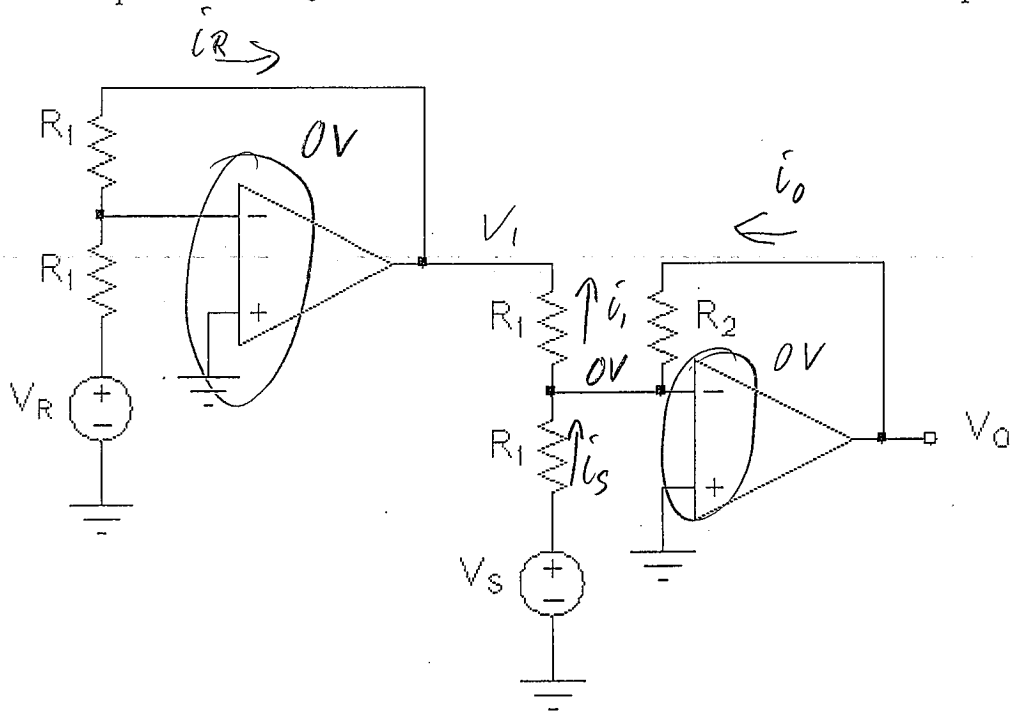
$$i_1 = \frac{V_S}{R_1} \Rightarrow V_O = V_S + R_2 \frac{V_S}{R_1} = \boxed{V_S\left(1 + \frac{R_2}{R_1}\right) = V_O}$$

$$i_S = \frac{V_S - V_O}{R} = \frac{V_S - V_S\left(1 + \frac{R_2}{R_1}\right)}{R} = V_S \frac{1 - 1 - \frac{R_2}{R_1}}{R} = V_S \frac{-R_2}{R_1 R}$$

$$\Rightarrow \boxed{\frac{V_S}{i_S} = -\frac{R_1 R}{R_2}}$$

Problem 5 (10 pts):

Find an expression for V_O as a function of the resistances and the two input voltages V_R and V_S:



$$i_R = \frac{V_R}{R_1} \Rightarrow V_1 = -i_R R_1 = -V_R$$

$$\Rightarrow i_1 = -\frac{V_1}{R_1} = \frac{V_R}{R_1}$$

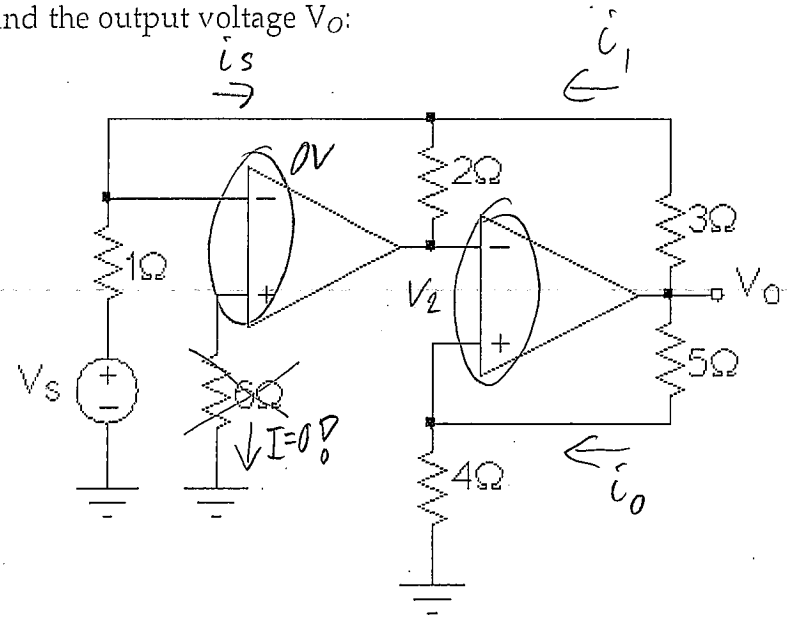
$$i_S = \frac{V_S}{R_1}$$

$$i_O = i_1 - i_S = \frac{V_R}{R_1} - \frac{V_S}{R_1} = \frac{1}{R_1}(V_R - V_S)$$

$$V_O = i_O R_2 = \boxed{\frac{R_2}{R_1}(V_R - V_S) = V_O}$$

Problem 6 ONLY 6610 students (10 pts):

Find the output voltage V_O:



$$i_s = \frac{V_s}{1\Omega} \qquad V_2 = -(i_s + i_1)2\Omega \qquad i_1 = \frac{V_0}{3\Omega}$$

$$V_0 - V_2 = i_0 5\Omega \qquad i_0 = \frac{V_2}{4\Omega}$$

$$V_0 - V_2 = \frac{V_2}{4\Omega} 5\Omega = \frac{5}{4}V_2 \Rightarrow V_0 = \frac{9}{4}V_2$$

$$V_2 = -\left(\frac{V_s}{1\Omega} + \frac{V_0}{3\Omega}\right)2\Omega \Rightarrow V_0 = -\frac{9}{4}\left(\frac{V_s}{1\Omega} + \frac{V_0}{3\Omega}\right)2\Omega$$

$$= -\frac{9}{2}\left(V_s + \frac{V_0}{3}\right)$$

$$V_0 + \frac{3}{2}V_0 = -\frac{9}{2}V_s$$

$$\frac{5}{2}V_0 = -\frac{9}{2}V_s \Rightarrow V_0 = -\frac{9}{5}V_s$$