

Write your name at the top right corner of every page (including this cover page).

Copy everything you want counted towards your grade onto the pages that I provided.

Write with a pen that cannot be erased!

No books or calculators are allowed!

Write down all the steps that lead to your result.

Identify new variables that you may introduce in the circuit diagrams that I provided.

Read all the problems before you start so that you can begin with those that seem easiest to you.

The last problem is for the Graduate Level students only. Undergraduate level students that solve the problem will get the earned points credited against their homework.

**Problem 1 (9 pts):**

In the table below please put a cross into the boxes where the condition stated for that row corresponds to the BJT condition stated for that column. Please leave open the boxes where that row condition about voltages and currents does not match the row condition of the BJT.

The <i>npn</i> BJT is:	active	saturated (ON)	cut-off (OFF)
$i_C = \beta i_B$	X		
$v_{BE} < 0.5\text{V}$			X
$v_{BE} \approx 0.7\text{V}$	X		
$i_C = i_B = i_E = 0$			X
$v_{BE} \approx 0.8\text{V}$ and $v_{CE} \approx 0.2\text{V}$		X	
$\beta i_B \geq i_C$		X	

Please write the three corresponding voltage conditions for a *pnp* BJT.

For *pnp* transistors we have:

$v_{BE} > -0.5\text{V}$  if the transistor is cut-off,

$v_{BE} \approx -0.7\text{V}$  in the active region

$v_{BE} \approx -0.8\text{V}$  and  $v_{CE} \approx -0.2\text{V}$  in saturation.

**Problem 2 (5 pts):**

Which of the following terms are allowed when you prepare your Transfer Function  $H(s)$  to sketch your Bode Plot? Mark “use” for those that are allowed, and “do not use” for those that are not allowed.

elementary form:	use	do not use
$\left(\frac{s^2+2\beta\gamma s+\gamma^2}{\gamma^2}\right)^{\pm 1}$	X	
$K$	X	
$\left(\frac{s+\alpha}{\alpha}\right)^{\pm 1}$	X	
$s^{\pm 1}$	X	
$\left(\frac{s^2-\delta^2}{\delta^2}\right)^{\pm 1}$		X

**Problem 3 (4 pts):**

Use only the straight-line asymptotes to sketch the Bode Plot (both, amplitude and phase!) for the transfer function:

$$H(j\omega) = \frac{j\omega}{a+j\omega} \text{ with } a > 0.$$

What type of filter is this?

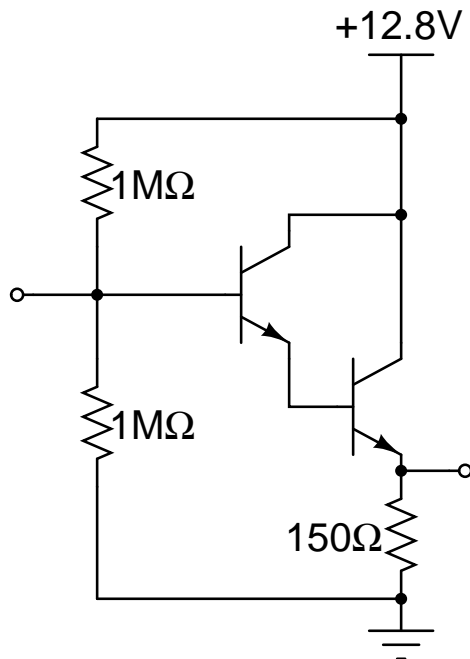
Solution: It is a high-pass filter. We express  $H(j\omega)$  as the product:

$$H(s) = \frac{1}{a} s \frac{a}{a+s}$$

The resulting Bode Plots show a phase that drops from +90 degree up to  $\omega = 0.1a$  to zero degree at  $\omega = 10a$ . Above  $\omega = 10a$  the phase stays at zero degree. The resulting dB plot rises through -20dB at  $\omega = 0.1a$  and hits 0dB at  $\omega = a$ . Above  $\omega = a$  it stays at 0 dB.

**Problem 4 (8 pts):**

For the Darlington configuration below, find  $i_{B1}$ , if the transistors both have  $\beta = 99$ .  $i_{B1}$  is the base current into the first transistor. The first transistor is the one that has the second transistor's base connected to its emitter.



Solution: Let us call the two  $1\text{M}\Omega$  resistors  $R_1$  and the  $150\Omega$  resistor  $R_2$ . The supply voltage shall be called  $V_{cc}$ . If  $i_1$  is the current through the upper  $R_1$  and  $i_2$  the one through the lower  $R_1$ , then:

$$i_1 = i_2 + i_{B1} \text{ and } V_{cc} = R_1(i_1 + i_2) = R_1(2i_1 - i_{B1}) \Rightarrow i_1 = \frac{1}{2}\left(\frac{V_{cc}}{R_1} + i_{B1}\right)$$

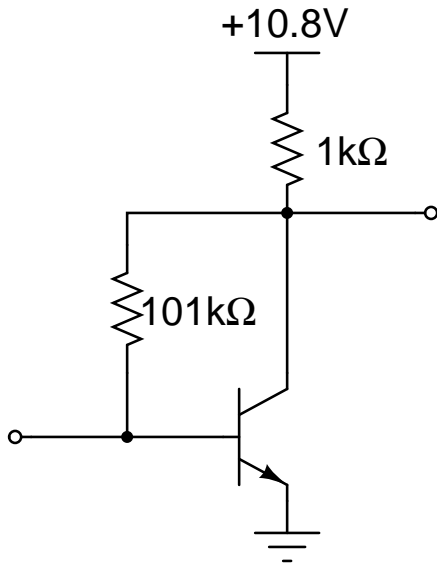
$$\text{also: } i_{E2} = i_{B2} + i_{C2} = i_{B2} + \beta i_{B2} = (\beta + 1)(i_{B1} + i_{C2}) = (\beta + 1)^2 i_{B1}$$

$$V_{cc} = i_1 R_1 + 0.7\text{V} + 0.7\text{V} + R_2 i_{E2}$$

$$\Rightarrow i_{B1}(R_1/2 + R_2(\beta + 1)^2) = V_{cc}/2 - 1.4\text{V} \Rightarrow i_{B1} = 2.5\mu\text{A}$$

**Problem 5 (6 pts):**

Below you find an example of collector feedback bias. Assuming that the transistor is operating in its active region and has  $\beta = 100$ , find  $i_B$ ,  $i_C$ , and  $v_{CE}$ . Confirm that the transistor is in the active region.



Solution: Let us call the supply voltage  $V_{cc}$  and the two resistors  $R_1$  and  $R_2$ , with  $R_1$  being the resistor that is connected to  $V_{cc}$ . Then:

$$V_{cc} = R_1(i_B + i_C) + R_2i_B + V_{BE} = R_1(101i_B) + R_2i_B + 0.7V$$

$$\Rightarrow i_B = \frac{V_{cc} - 0.7V}{101R_1 + R_2} = 50\mu A \Rightarrow i_C = 5mA$$

$V_{CE} = V_{cc} - R_1(i_B + i_C) = 10.8V - 5.05V = 5.75V$ , which also proves that the transistor is in the active region.

**Problem 6 (3 pts):**

Put the following two sinusoids into phasor form and add the phasors:

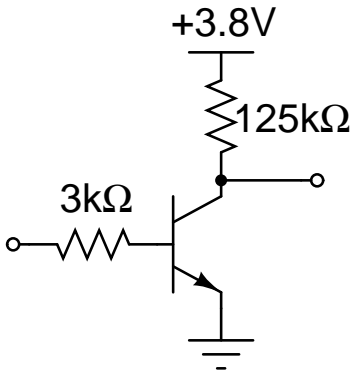
$$i_1(t) = 20A \cos(\omega t + 90^\circ)$$

$$i_2(t) = 10A \sin(\omega t + 180^\circ)$$

Solution:  $j20 + j10 = j30$ ; the angles are the same in both cases, as  $\sin(180) = \cos(180-90) = \cos(90)$ , i.e. 30A with an angle of +90 degree

**Problem 7 ONLY 6610 students !!! (6 pts):**

Below you find a simple logical inverter. So when its input goes high (3.8V), the transistor should saturate as to have the minimum possible output voltage. What is the minimal  $\beta$  the transistor must have to saturate at an input voltage of 3.8V given the resistors as shown?



Solution: If we call the collector resistor  $R_C$ , the base resistor  $R_B$ , and the supply voltage  $V_{cc}$ , in saturation we will have:

$$i_C R_C = V_{cc} - 0.2V = \beta i_B R_C$$

The input current will be:

$$i_B R_B = V_{cc} - 0.8V$$

Therefore:

$$\beta = \frac{i_C}{i_B} = \frac{R_B}{V_{cc} - 0.8V} \frac{V_{cc} - 0.2V}{R_C} = \frac{3.6V}{3.0V} \frac{3k\Omega}{125k\Omega} = 0.03$$

P.S. My mistake here: The values for  $R_C$  and  $R_B$  are swapped in the figure... (Would have been a wonderful  $\beta = 50$  if it wasn't - and the promised inverter...)