

Flux Compactification Geometries & de Sitter in Heterotic M-Theory

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Overview:

- Introduction (Moduli Stabilization, Heterotic M-Theory)
- The Linear Flux Background
- de Sitter Vacua from Moduli Stabilization in the Linear Background: Getting Minima
- The Exact Flux Background
- de Sitter Vacua from Moduli Stabilization in the Exact Background: Global Potential Landscape

Introduction

Vacuum Selection Problem in string-theory compactifications:

though string-theory in $D=10$ has only one free parameter, the string-tension, compactifications of string-theory on CY manifolds which lead to $D=4$, $N=1$ Susy give large proliferation of vacua ($\approx 10^6$ for heterotic string)

4d 'fundamental' constants such as G_N or gauge couplings depend on compactification moduli

⇒ **problem of moduli stabilization**: in any realistic compactification all moduli have to be fixed (or at least very slowly rolling)

⇒ Stabilization can be achieved by **non-perturbative physics** and/or **fluxes**

Early attempts to use non-perturbative world-sheet instanton effects (Dine, Seiberg, Witten '86, '87) led to **runaway behavior**, i.e. radius of compactification manifold runs to large values giving **decompactification**

However, M-Theory and in particular Heterotic M-Theory exhibits further **non-perturbative** contributions and fluxes which have to be included as well

Type IIB Compactifications:

superpotentials

$$W = \int_{CY} G_3 \wedge \Omega$$

resulting from non-vanishing $G_3 = H_{RR} - \tau H_{NS}$ typically lead to **stabilization of all complex structure moduli and dilaton** (Giddings, Kachru, Polchinski '01)

Recently (Kachru, Kallosh, Linde, Trivedi '03) non-perturbative D3 instanton / GC were included to stabilize also the **Kähler moduli** and were led to D=4 supersymmetric AdS vacua.

By breaking supersymmetry through the addition of an additional $\overline{D3}$ -brane KKLT obtain D=4 dS vacua

Our focus here will be on **heterotic M-theory**:

Why Heterotic?

to have realistic gauge groups and matter included right from the start (adding them through branes later might severely alter the geometry and therefore the mechanism to obtain de Sitter)

Why M-Theory?

stabilization in particular of the dilaton in weakly coupled heterotic string with H-flux and GC generically leads one to strong coupling. Therefore we are working in (heterotic) M-theory which in addition has important phenomenological advantages as compared to the weakly coupled case (Newton's Constant, gauge and gravitational unification by lowering the string scale, gaugino masses)

Heterotic M-Theory

1. Picture: Set-Up

Theory **requires fluxes** in a specific form (they are not optional and determined through the boundary and M5-brane sources). \exists 3 types of fluxes

$$G_{(2,2,0)} \quad G_{(0,3,1)} = H_{0,3} \quad G_{(1,2,1)} = H_{1,2}$$

in this talk we will mostly ignore the latter two

note: the H fluxes are localized on the $D=10$ boundaries while only the $G_{(2,2,0)}$ component lives in the $D=11$ bulk and has a direct influence on the geometry there

the first two types of fluxes lead to a **warped compactification manifold** conformal to a CY – thereby moduli stabilization can be studied without having to deal with **non-Kähler manifolds** (3rd type of flux) whose moduli are still largely unknown.

- $G_{(2,2,0)}$ will warp the geometry in x^{11} (orbifold) direction and is important for the stabilization of the orbifold length (**dilaton**) in combination with non-perturbative effects (Curio, A.K. '01)
- $H_{0,3}$ is important for the stabilization of the **complex structure moduli** (Gukov, Kachru, Liu, McAllister '03)
- $H_{1,2}$ stabilizes the CY **volume modulus** (Becker, Becker, Dasgupta, Green '03)

The Linear Flux Background

To connect to D=4 one has to compactify Het M on

$$M_6 \times S^1/Z_2$$

To preserve D=4, N=1 Susy M_6 will turn out to be **conformal** ($\rightarrow G_{(2,2,0)}$) to a **Calabi-Yau threefold** (Witten '96)

Derivation of **supersymmetric geometry** via Killing-Spinor equ:

$$\delta\Psi_I = \left(D_I + \mathcal{G}_I \right) \eta \stackrel{!}{=} 0$$

where

$$D_I \eta = \left(\partial_I + \frac{1}{4} \Omega_{IJK} \Gamma^{JK} \right) \eta$$

$$\mathcal{G}_I \eta = \frac{\sqrt{2}}{288} \left(\Gamma_{IJKLM} - 8g_{IJ} \Gamma_{KLM} \right) G^{JKLM} \eta$$

$G \neq 0 \Rightarrow D_m \eta \neq 0$, i.e. Deformation of initial CY-Geometry

Concentrate on $G_{(2,2,0)}$ flux whose visible boundary source can be characterised through the charge

$$\begin{aligned} S_v &= \frac{1}{8\sqrt{2}} \omega^{lm} \omega^{np} G_{v,lmnp} \\ &\simeq -\frac{\kappa^{2/3}}{V_v} \int_{CY} \omega \wedge (\text{tr} F \wedge F - \frac{1}{2} \text{tr} R \wedge R)_v \\ &\simeq \frac{\kappa^{2/3}}{V_v} V_v^{1/3} > 0 \end{aligned}$$

Linearized Warped Background Ansatz (Witten '96):

$$ds^2 = (1 - f_{lin}(x^n, x^{11}))\eta_{\mu\nu}dx^\mu dx^\nu + (1 + f_{lin}(x^n, x^{11}))(g_{lm}(x^n)dx^l dx^m + dx^{11}dx^{11})$$

where

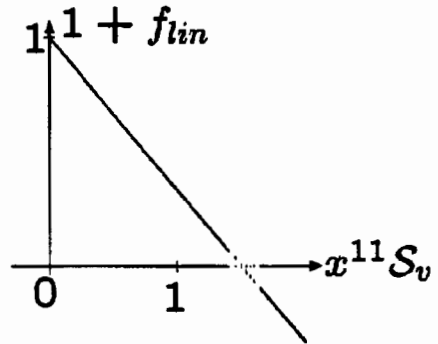
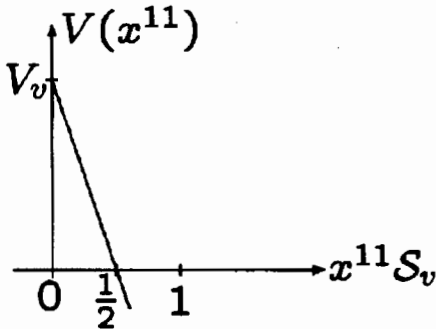
$$f_{lin} \ll 1$$

⇒ Solution:

$$f_{lin} = -\frac{2}{3}x^{11}S_v$$

⇒ Volume of deformed Calabi-Yau:

$$V(x^{11}) = V_v(1 - 2x^{11}S_v)$$



⇒ approximation remains valid as long as one obeys

$$\epsilon \simeq \left(\frac{\kappa}{V_v}\right)^{2/3} L \ll 1$$

The D=4, N=1 Effective Supergravity

D=4, N=1 effective SUGRA results from a reduction of the D=11 Het M over this warped flux compactification background (Lukas, Ovrut, Waldram '97)

⇒ it therefore inherits the requirement for **small ϵ**

Gauge field background: e.g. by standard embedding of the spin connection ($SU(3)$ holonomy) into the visible boundary E_8 gauge connection one breaks the visible E_8 down to E_6 . I.e. visible boundary matter fields C in the fundamental **27** of E_6 . In standard embedding the hidden gauge group remains E_8

The (dimensionless) **universal moduli**

\mathcal{V} = normalized average CY volume

\mathcal{L} = normalized orbifold length

$x \in [0, 1]$ = position of M5 brane along orbifold

are combined into the chiral superfields

$$S = \mathcal{V} + i\sigma_S$$

$$T = \mathcal{V}_{OM} + i\sigma_T$$

$$Z = \mathcal{V}_{OM}x + i\sigma_Z$$

(σ_I universal axions) plus matter C where

$$\mathcal{V}_{OM} = \mathcal{L} \left(\frac{6\mathcal{V}}{d} \right)^{1/3} = \text{volume of OM instanton}$$

Dimensionless moduli \mathcal{V} , \mathcal{L} are related to dimensionful CY volume $V(x^{11})$ and orbifold length L through

$$\mathcal{V} = \frac{1}{vL} \int_0^L dx^{11} V(x^{11}), \quad \mathcal{L} = L/l, \quad z = x_5^{11}/L$$

where v and l are two dimensionful reference values

$$v = 8\pi^5 l_{11}^6, \quad l = 2\pi^{1/3} l_{11}$$

D=4, N=1 effective supergravity specified by

A) Kähler-potential

$$K = K_{S,Z} + K_T + K_C + K_{cx} + K_{bd},$$

where

$$K_{S,Z} = -\ln \left(S + \bar{S} - \frac{(Z + \bar{Z})^2}{(T + \bar{T})} \right)$$

$$K_T = -\ln \left(\frac{d(T + \bar{T})^3}{6} \right)$$

$$K_C = \left(\frac{3}{T + \bar{T}} + \frac{2\xi}{S + \bar{S}} \right) H_{IJ} C^I \bar{C}^J + \mathcal{O}(C^3)$$

$$K_{cx} = -\ln \left(-i \int_{CY} \Omega \wedge \bar{\Omega} \right)$$

and $K_{bd} \simeq 10^{-5} K_{S,Z}, K_T$ (Buchbinder, Ovrut '03)

B) perturbative Superpotential

$$W_p = \frac{4\pi\sqrt{2}}{3} \lambda_{IJK} C^I C^J C^K$$

Yukawa couplings are quasi-topological, i.e. depend on the complex structure and bundle moduli. We won't need the gauge kinetic function

Non-perturbative Contributions

- **OM's** stretching from boundary to boundary (Heterotic Worldsheet Instantons) \Rightarrow often give zero by summing over all the different holomorphic curves contained in a given homology class
- **OM** which stretch between one boundary and the **M5** (Moore, Peradze, Saulina '00) and wrap an internal hol. 2-cycle Σ

$$W_{OM} = h(e^{-Z} + e^{Z-T})$$

OM favours large Orbifold-Lengths L

- **Gaugino Condensation** (Horava '96, Lukas, Ovrut, Waldram '97) appears naturally on the strongly coupled hidden boundary

$$W_{GC} = -C_H \mu^3 e^{-\frac{1}{c_H}(S-\gamma T)}$$

where μ is cut-off scale at which hidden gauge theory is defined and $\gamma = S_v L \simeq 1$. GC favors small L

- **M5-instantons** (Becker, Becker, Strominger '95) which wrap the whole CY

$$W_{M5} = h' e^{-S/4}$$

are additional sources of G-flux leading to jumps in volume $V(x^{11})$. Except for this x -position dependence they behave similar to GC

For the linearized background one has to use

$$\epsilon \ll 1 \quad \Rightarrow \quad \mathcal{V} \gg \mathcal{V}_{OM}$$

\Rightarrow **OM instantons are most dominant non-perturbative effects**

Minima with Positive Vacuum Energy

Concentrate on

- single parallel M5-brane at position

$$x_5^{11} = xL, \quad x \in [0, 1]$$

which is D=4 spacetime-filling and wraps once a basis holomorphic 2-cycle Σ of the CY (Susy preserving)

- Take Σ to be isolated
⇒ no integration over moduli specifying its position within the CY
- choosing $h^{1,1}(CY) = 1$ (e.g. a Quintic)
⇒ one T -modulus

Get the **Potential** from standard D=4, N=1 SUGRA expression

$$\frac{U_{OM}}{M_{Pl}^4} = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2)$$

Leading plus sub-leading contributions to potential w.r.t. expansion in ϵ come from

$$\frac{U_{OM}}{M_{Pl}^4} = e^K K^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W} + \dots$$

⇒ **Moduli Potential** (Moore, Peradze, Saulina '00; Curio, A.K. '01)

Minimization w.r.t. axion-sector of the theory gives

$$2\sigma_Z = \sigma_T \text{ mod } 2\pi$$

⇒ Potential including subleading order becomes

$$\frac{U_{OM}}{M_{Pl}^4} = \frac{3|h|^2 e^{K_{\alpha\alpha}}}{4d\mathcal{V}_{OM}^2} \left\{ \left[e^{-\mathcal{V}_{OM}x} - e^{-\mathcal{V}_{OM}(1-x)} \right]^2 + \frac{2\mathcal{V}_{OM}}{3\mathcal{V}} \left[e^{-\mathcal{V}_{OM}x}x + e^{-\mathcal{V}_{OM}(1-x)}(1-x) \right]^2 \right\}$$

⇒ **positive vacuum energy**

⇒ $x = 1/2$ minimizes U_{OM} , i.e. **M5-brane** assumes a **symmetric position** in the middle of the orbifold-interval

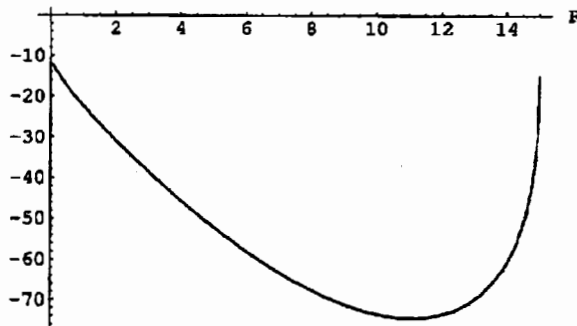
⇒ potential's leading term vanishes and one gets

$$\frac{U_{OM}}{M_{Pl}^4} = \frac{|h|^2 e^{K_{\alpha\alpha}}}{2d\mathcal{V}_{OM}\mathcal{V}} e^{-\mathcal{V}_{OM}} \geq 0$$

⇒ if \mathcal{V}_{OM} and \mathcal{V} were independent, then running towards decompactification limit $\mathcal{V}_{OM} \rightarrow \infty$.

But in the presence of $G_{(2,2,0)}$ flux also \mathcal{V} depends non-trivially on \mathcal{L}

⇒ Minimization w.r.t. \mathcal{L} leads to **minimum**



[$\ln(U_{OM}/M_{Pl}^4)$ as a function of the orbifold modulus $R \propto \mathcal{L}$ for parameters $|h| = 1, K_{cx} = K_{bd} = 0, \mathcal{V}_v = 3000, 3S_5 = 800/(\mathcal{V}_v l), d = 30$]

⇒ vacua which **break supersymmetry spontaneously** through non-vanishing F-terms

Avoiding negative CY volume before and behind the M5 brane fixes the moduli values at the minimum at

$$L = \frac{1}{S_v} \quad \rightarrow \quad V(L) = \frac{V_v}{4}$$

and requires **flux equality**

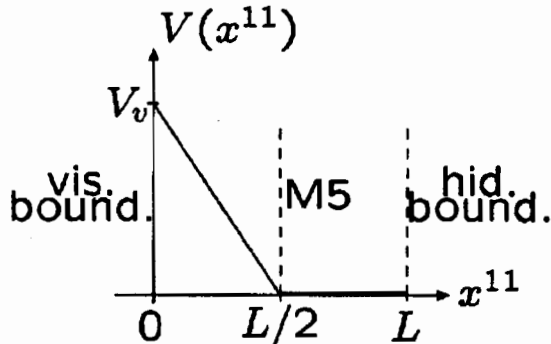
$$S_v + S_5 = 0 \quad \Rightarrow \quad S_h = 0$$

which leads to a difference in the 'instanton-numbers' on both boundaries caused by the flux-jump of the M5

$$\begin{aligned} \int_{CY} \omega \wedge \text{tr} F_h^2 - \int_{CY} \omega \wedge \text{tr} F_v^2 &= 8\pi^2 \int_{\Sigma} \omega \\ &= 8\pi^2 \int_{CY} \omega \wedge [\Sigma] \propto W_G \end{aligned}$$

D=11 Geometry of Vacua

D=11 Geometry of these vacua (using minimizing \mathcal{V}, \mathcal{L} plus flux-equality):



To lift the zero half-interval one might invoke quantum corrections but there is an easier cure!

⇒ One cannot study the potential towards the right of the minimum as here the CY volume becomes negative!

⇒ To study the potential globally one has to go beyond the approximative linearized flux background which we will do now in the second part of the talk

Exact Flux Compactification Geometry

(G. Curio, A.K. '00, '03)

Solve Killing-Spinor Equ. exactly incorporating all non-linear field-theory corrections demanded by D=11 Susy. Warped Geometry Ansatz (for simplest case w/ only $G_{(2,2,0)} \neq 0$):

$$ds^2 = e^{-f(x^n, x^{11})} \eta_{\mu\nu} dx^\mu dx^\nu + e^{f(x^n, x^{11})} (g_{lm}(x^n) dx^l dx^m + dx^{11} dx^{11})$$

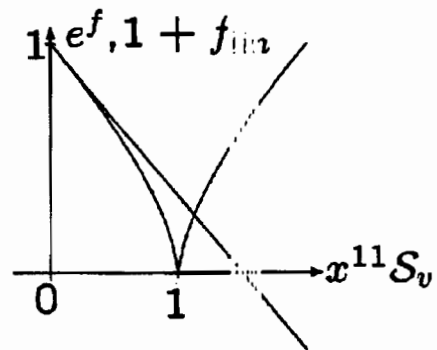
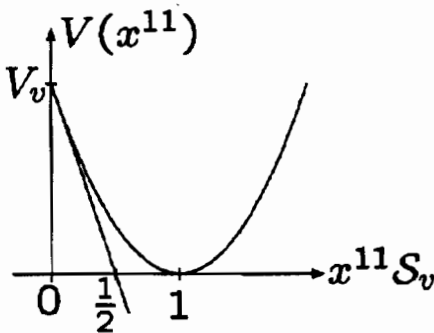
w/o assumption that f is small expansion parameter

⇒ Result:

$$e^{f(x^{11})} = |1 - x^{11} S_v|^{2/3}$$

⇒ Volume of deformed Calabi-Yau:

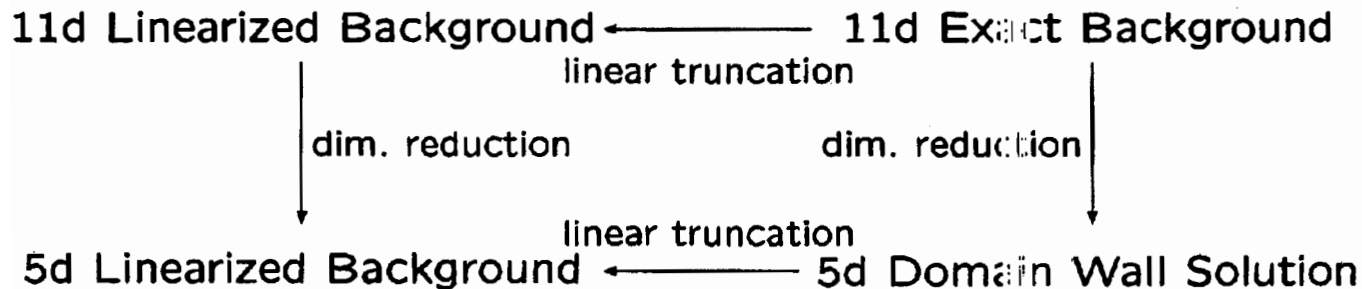
$$V(x^{11}) = V_v (1 - x^{11} S_v)^2$$



⇒ **extends** Witten's linearized geometry and recovers it as the **tangent** approximation at the visible boundary

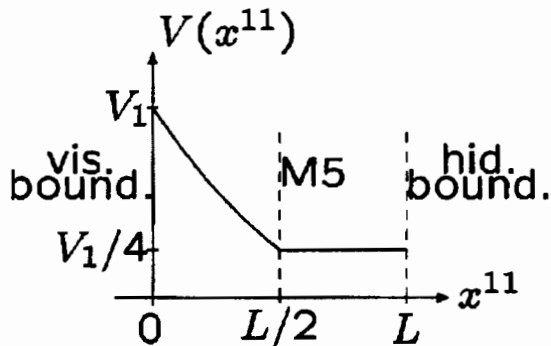
⇒ **solves** problem with **negative metric and CY-volume**. Quantum corrections by R^4 terms may give a small positive shift in the volume if the Euler-number of the CY is positive $\chi > 0$ (Strominger '98). This could lift the localized naked singularity

The connections between various heterotic M-theory backgrounds geometries in 11d and 5d:



D=11 Geometry in Full Background

With same data $x = 1/2$ and flux-equality, $S_v + S_5 = 0$ the exact warped background geometry leads to the following situation at the critical point



⇒ While in the linearized background extending L beyond its stabilized value $L = 1/S_v$ leads to negative CY volume and prohibits therefore the **global study of the moduli potential** there is no problem in doing so when the exact background is used (CY volume non-negative everywhere!)

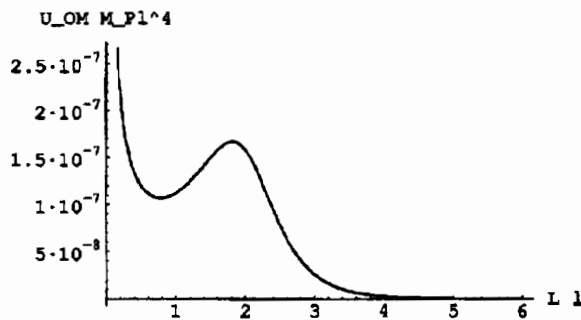
The Global View: de Sitter and Runaway

(A.K. '04)

Minimization procedure same as before with the difference that now we have to derive the average CY volume \mathcal{V} and \mathcal{V}_{OM} from the exact background geometry instead of the linearized one

⇒ this changes the L dependence of the moduli potential when L is no longer small as then the exact background differs drastically from the linearized one.

Result for $x = 1/2$, $S_v + S_5 = 0$:



[U_{OM}/M_{Pl}^4 as a function of the orbifold modulus $\mathcal{L} = L/l$ for parameters $|h| = 1, K_{ex} = K_{bd} = 0, \nu_v = 450, S_0 = 500/(\nu_v l), d = 10^5$]

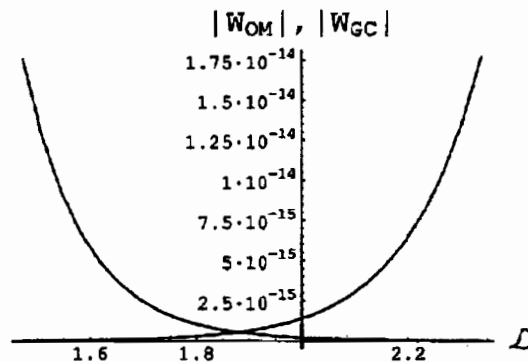
However, to obtain minimum typically $d \geq 10^4$ required and one obtains $\mathcal{V}_{OM} < 1$. Therefore minimum lies in a regime where supergravity is not under good control and moreover multiply wrapped OM's can potentially give large corrections.

De Sitter from Balancing OM against GC

(M. Becker, G. Curio and A.K. '04)

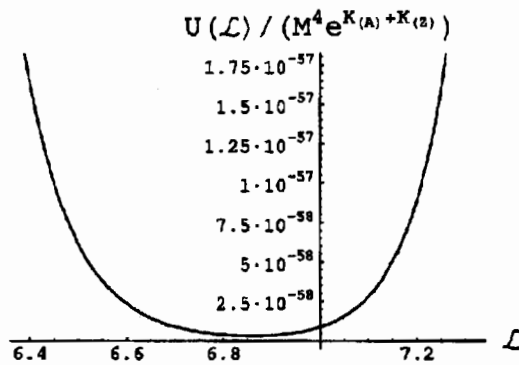
Observation: in exact background OM-instantons are no longer the only dominant non-perturbative effect as there is no requirement any longer to have a perturbatively small $\epsilon \ll 1$ (this was only crucial for the linearized background). Thus **gaugino condensation** GC is a priori no longer hierarchically smaller than OM's and has to be included. This is good news for the orbifold length (dilaton) stabilization in the full exact background as GC shows different monotonicity properties wrt \mathcal{L} as compared to OM

$$|W_{OM}| \sim |h|e^{-\nu_{OM}}, \quad |W_{GC}| \sim |g|e^{-\frac{1}{c_H}(\nu - \gamma\nu_{OM})}$$



$|W_{OM}|$ (left curve) and $|W_{GC}|$ (right curve)

This **balancing** between OM and GC indeed leads to a robust minimum of the potential



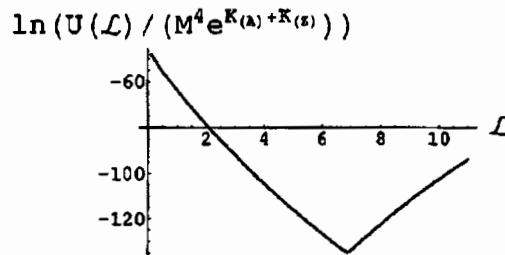
the resulting moduli potential as a function of \mathcal{L}

Results:

- **positive vacuum energy** local dS minima
- now they lie in a regime where $\mathcal{V}_{OM}, \mathcal{V} \gg 1$, i.e. where **supergravity is under good control** and where the CY intersection numbers d need no longer assume large values
- $D_S W = -\frac{W_{GG}}{C_H} \neq 0 \rightarrow$ **supersymmetry is broken spontaneously through F-terms**
- not only the dilaton (\mathcal{L}) is stabilized but also S and T axions. Moreover visible charged matter fields C **receive a non-vanishing vev** and are **no longer massless** as in the heterotic string case! This vev acquires geometric exponential suppression factor (balancing between C and OM+GC at minimum) which allows to bring the vev down to TeV region with moderate $\mathcal{V}_{OM}, \mathcal{V}$

The Global View

The global potential as a log-plot:



the logarithm of the moduli potential

- from log-plot clearly recognizable: balancing between OM and GC – dominance of OM to lhs while dominance of GC to rhs of minimum
- at fixed visible CY volume \mathcal{V}_v , there is an **upper bound on \mathcal{L}** given through the position \mathcal{L}_{max} of the naked singularity. This value is determined through the visible boundary charge

$$\mathcal{L} \leq \mathcal{L}_{max} = \frac{1}{S_v l} \propto \mathcal{V}_v^{1/3}$$

In **decompactification limit**: $\mathcal{V}_v \rightarrow \infty$, therefore also upper bound $\mathcal{L}_{max} \rightarrow \infty$. Then

$$|W_{GC}| \sim e^{-\mathcal{V}_v/C_H} \rightarrow 0, \quad |W_{OM}| \sim e^{-\mathcal{L}\mathcal{V}_v^{1/3}} \rightarrow 0$$

and vacuum energy is converging towards zero in decompactification limit. Therefore dS vacua are as expected **metastable**.

Some Features of the de Sitter Vacua

- Hierarchy of F-terms

$$|D_S W| \sim |D_T W| \gg |D_\alpha W| > 0 \rightarrow F^{\bar{S}} > F^{\bar{T}} \gg F^{\bar{\alpha}}$$

F-term related to S causes supersymmetry breaking

- **supersymmetry breaking scale**

M_{SUSY} given by the largest vev of the F-terms

$$\begin{aligned} M_{SUSY} &= M \max \{|F^{\bar{i}}|^{1/2}\} = M |F^{\bar{S}}|^{1/2} \\ &= M e^{\frac{1}{4}(K_{(A)} + K_{(z)})} \left(\frac{6\mathcal{V}_0^3}{d\mathcal{V}_{OM,0}^3} \right)^{1/4} \left(\frac{|W_{GC}|}{C_H} \right)^{1/2} \end{aligned}$$

- **gravitino mass**

$$\begin{aligned} m_{3/2}^2 &= M^2 e^{K/2} |W| \\ &= M e^{\frac{1}{4}(K_{(A)} + K_{(z)})} \left(\frac{3}{8d\mathcal{V}_0 \mathcal{V}_{OM,0}^3} \right)^{1/4} (|W_{OM}| + |W_{GC}|)^{1/2} \end{aligned}$$

Hence ratio of supersymmetry breaking scale to gravitino mass

$$\frac{M_{SUSY}}{m_{3/2}} = \frac{2\mathcal{V}_0}{C_H^{1/2}} \left(\frac{|W_{GC}|}{|W_{OM}| + |W_{GC}|} \right)^{1/2}$$

Dependence on Hidden Gauge Group

The values

$$E = (U/e^{K(z)})^{1/4}, \quad \tilde{M}_{SUSY} = M_{SUSY}/e^{K(z)}, \quad \tilde{m}_{3/2} = m_{3/2}/e^{K(z)}$$

are given in the following table at the critical point. Notice the factor $e^{-K(z)}$ as compared to the actual physical parameters $U^{1/4}, M_{SUSY}, m_{3/2}$

H, C_H	$E_8, 30$	$E_6, 12$	$SO(10), 8$	$U(5), 5$	$SU(3), 3$
\mathcal{L}_0	1.1	3.0	4.1	5.5	7.1
\mathcal{L}_{max}	7.2	7.2	7.2	7.2	7.2
V_0	341	256	215	171	135
$V_{OM,0}$	7	16	21	26	31
E/TeV	1.2×10^9	5.7×10^6	495213	36455	3185
M_{SUSY}/TeV	3.1×10^{10}	1.1×10^8	8.5×10^6	27697	39158
$\tilde{m}_{3/2}/TeV$	3.4×10^8	1.3×10^6	103752	6991	567

The dependence on the hidden gauge group H which enters through its dual Coxeter number C_H

Result: Hidden Gauge Group of small rank favored since

- it brings \tilde{M}_{SUSY} , etc. close to the **TeV regime**
- it leads to stabilization of hidden boundary close to \mathcal{L}_{max} . This leads to the **correct 4d Newton's Constant** (Witten '96, Curio, A.K. '03)
- once the hidden E_8 is broken one has hidden matter which serves as an excellent candidate for **dark matter** (interacts with visible matter or gauge fields only (super-)gravitationally and can be expected to exhibit similar clustering as visible matter