

Geometric Transitions,

Flops and Non-Kähler

Manifolds

Radu Tatar (Berkeley)

Snowbird, June 2004

Melanie Becker, Keshav Dasgupta,
Anke Knauf, RT, hep-th/0403288
in preparation

①

Usual story:

II A / II B compactification
on CY manifolds



$N=2$ SUSY in 4-dimensions



extra fluxes or/and D-branes
 $N=1$ SUSY in 4-dimensions

Calabi - Yau: $\left[\begin{array}{l} d\mathcal{J} \neq 0 \text{ (Kähler form)} \\ d\Omega^{(3,0)} \neq 0 \text{ (complex)} \\ SU(3) \text{ holonomy} \end{array} \right.$

Non
Calabi - Yau:

$\left[\begin{array}{l} SU(3) \text{ structure} \\ d\mathcal{J} \neq 0 \text{ (non-Kähler)} \\ d\Omega \neq 0 \\ \text{breaks } 1/4 \text{ of SUSY} \end{array} \right.$

① Motivation for Studying Compactifications on Non-Kähler Manifolds:

① They appear as possible compactifications for heterotic strings (Strominger)

K. Becker, M. Becker, K. Dasgupta, P. S. Green
E. Sharpe
hep-th/0209077, 0301161, 0310058, 0312221

D. Lust et al.

hep-th/0211118, 0306088, 0310021

② $\text{II B} / \text{II A}$ compactifications on Non-Kähler can be characterized by torsion classes and they are related to M theory compactified on G_2 manifolds with torsion.

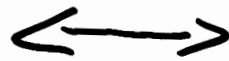
③ As I shall show, they are required for a "fine-tuning" of the concept of "Geometric Transitions" to keep track of the NS fields.

②

④

As I shall clarify, there is a T-duality relation between

$\underline{\text{II}} B$ Compactified
on
CY with RR
and NS fluxes



$\underline{\text{II}} A$ Compactified
on non-Kähler
with fluxes

We now have a nice "Landscape picture" proposal for $\underline{\text{II}} B$ compactification in the presence of fluxes.

It would be nice to be able to have a check by considering a related theory and $\underline{\text{II}} A$ on non-Kähler is that "theory"

There are other directions, I shall mention them in the course of my talk.

②

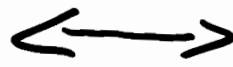
④

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Outline of my talk

- 1) As the initial motivation was to clarify some aspects of the geometric transition, I shall start with a review:
- tools for obtaining strong coupling results in effective field theories.
 - allow tests of open/closed string dualities

2) Concrete Example (Vafa 0008142)

D6 brane on S^3 inside deformed conifold

\Updownarrow dual (open/closed sense)

$\left. \begin{array}{l} \text{NS} \\ \text{RR} \end{array} \right\}$ Fluxes on resolved conifold

The actual results of Vafa: RR fluxes on a resolved conifold

with $d\Omega \neq 0$

\downarrow

NS 4-form

④

Our "New Transition":

D6 branes on S^3 inside a non-Kähler deformation of Deformed Conifold

\Updownarrow open/closed string

RR fluxes on a non-Kähler deformation of the Resolved Conifold

3) Next step: lift to 11 dimensions

D6 branes on S^3
inside non-Kähler

\rightarrow

Manifold with
 G_2 structure

(torsional
 G_2 manifolds)

\downarrow flops + reduction

RR fluxes on
non-Kähler manifolds

4) Conclusions

⑤

Geometric Transitions - based on

Open/closed string dualities
involving topological strings:

"Open topological string on T^*S^3
with N topological D-branes



closed topological strings
on resolved conifold"

Chern-Simons on S^3

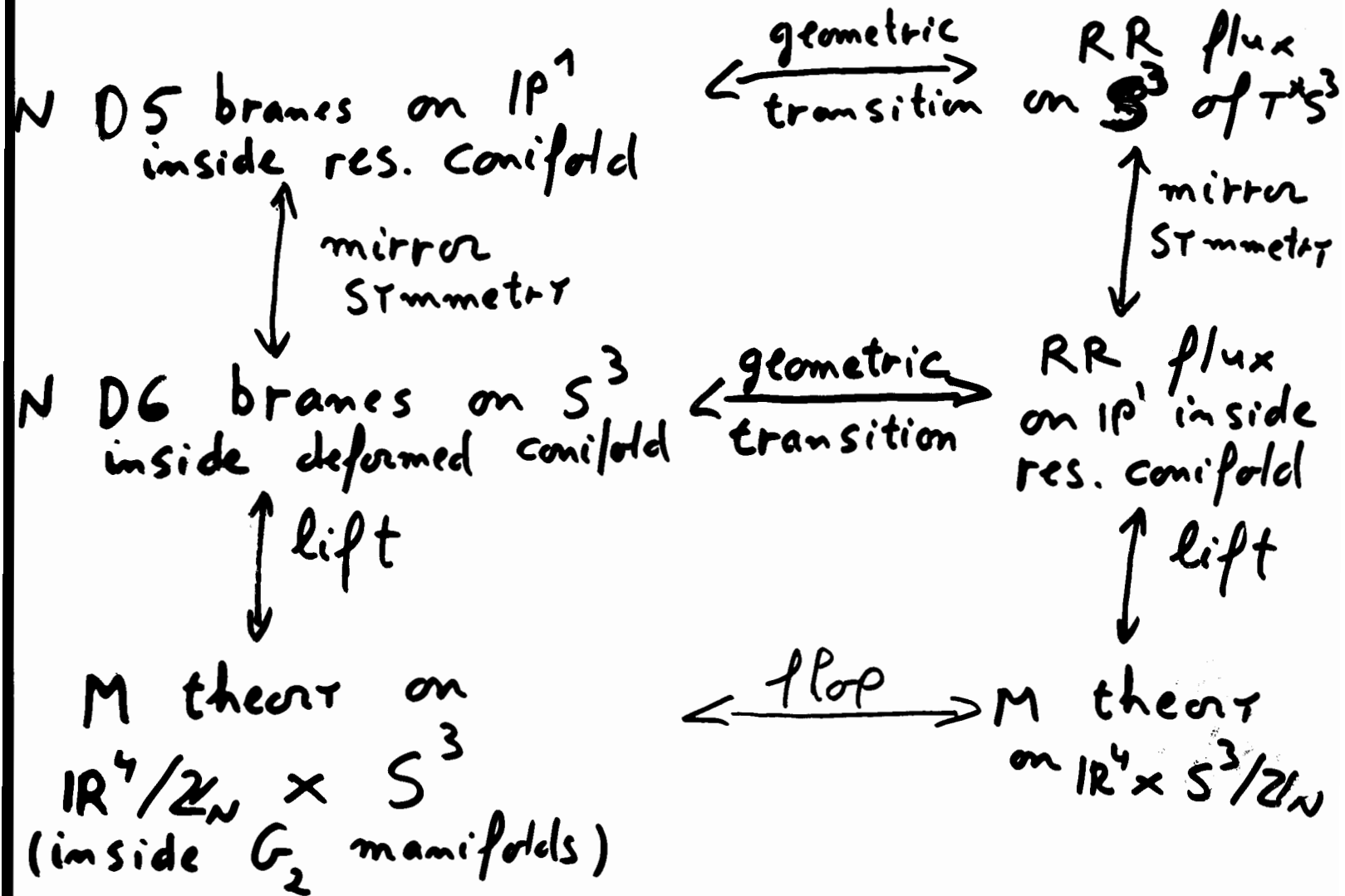
t - size of IP^1

Partition functions are equal for

$$t = i g_s N$$

Vafa (0008142) embedded topological string results into Superstrings.

The duality web becomes:



⑦

What are the maps?

II B:

N D5 branes on IP^1 \longleftrightarrow H_{RR} on S^3
 g -coupling ct. for $U(N)$

$$\int_{S^3} H_{RR} = N$$

$$\int_{IR^3} \tau H_{NS} = \frac{1}{g^2}$$

Crucial condition

gluino condensate = size of S^3

$$S = \text{size of } S^3 = \int_{S^3} \Omega^{(3,0)}$$

II A:

N D6 branes on S^3 \longleftrightarrow $B_{RR}^{(2)}$ on IP^1

$$S = \text{volume of } IP^1$$

But there is an extra "surprise" for II A!

⑧

Explicit computations of genus 0 open topological string amplitudes (BCOV):

$$W = \sum_h \int d^2\theta F_{0,h} [N h S^{h-1}] + \alpha S_{h=2}$$

part. funct. for topological strings at $g=0, h$ holes

⇓

We need a term in the closed string

$$W \sim t$$

complex volume of IP^1

This appears from the integral:

$$W = \int_{res. conif} t \wedge \Omega^{(4)} - \text{geometric 4-form}$$

So we need an extra $\Omega^{(4)}$.

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Proposal:

$$\left. \begin{aligned} \Omega^{(4)} &= d\Omega_{\text{II}A}^{(3,0)} \\ \text{and} \\ d\mathcal{J} &\neq 0 \end{aligned} \right\}$$

the resolved
conifold becomes
non-Kähler and
non-complex

The RR 2-form is the same
There is an extra NS field which
combines with \mathcal{J} to give

$$\int (\mathcal{J} + i B_{NS}) \wedge \Omega^{(4)}$$

And we also change the geometric
transition setup, by considering open
string side also compactified on a
non-Kähler manifold.

We now proceed to build
an explicit example.

(10).

Start in II B : solution for D5
branes wrapped on \mathbb{P}^1 inside res. conifold

$$dS_{10}^2 = h^{-1/2}(\rho) dx^\mu dx_\mu + h^{1/2}(\rho) dS_6^2$$

$$dS_6^2 = \gamma' dr^2 + \frac{1}{4} \gamma \sum_{i=1}^2 (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) +$$

$$+ \frac{1}{4} \gamma' r^2 \left(d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 +$$

$$+ a^2 (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \quad \left(\begin{array}{l} \text{Pando-Zaras} \\ \text{Tsertlin} \end{array} \right)$$

a - size of \mathbb{P}^1

h - harmonic function of ρ

$$B_{NS} = \int_1(\rho) d\theta_1 \wedge d\phi_1 + \int_2(\rho) d\theta_2 \wedge d\phi_2$$

$H_{NS} = dB_{NS}$ is very small such
that there is no
backreaction on the
metric

①

We make the following change of coordinates:

$$(\phi_1, \phi_2, \psi) \rightarrow (x, y, z)$$

$$(dx, dy, dz) = \left(\frac{1}{2} \sqrt{h^{y_2} \gamma} \sin \theta_1 d\phi_1, \right.$$

$$\left. \frac{1}{2} \sqrt{h^{y_2} (\gamma + 4a^2)} \sin \theta_2 d\phi_2, \right.$$

$$\left. \frac{1}{2} r \sqrt{\gamma'} h^{y_2} d\psi \right)$$

in the new coordinates

$S_1^2(\phi_1, \theta_1)$ are converted into ~~tori~~ tori

$S_2^2(\phi_2, \theta_2)$

Metric on the tori:

$$|dz_i|^2 = |dx - \tau_i d\theta|^2$$

complex structure

$$\tau_1 = \frac{i}{2} \sqrt{\gamma' h}$$



Strominger - Yau - Zaslow:

For a manifold admitting a T^3 structure, one can reach a mirror by T-duality on T^3

Mirror Symmetry:
replacing Kähler deformations by
Complex Deformation

For a conifold

$$xy - uv = 0 \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} \quad \left. \begin{matrix} x \}_{_1 = u \}_{_2 \\ y \}_{_2 = v \}_{_1} \\ h_{11} = 1 \end{matrix} \right) \begin{matrix} \text{Resolved} \\ \text{conifold} \\ \begin{matrix} \}_{_1 \rightarrow \mathbb{P}^1 \\ \}_{_2 \end{matrix} \end{matrix}$$

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \quad \begin{matrix} xy - uv = M \\ x_1^2 + x_2^2 + y_1^2 + y_2^2 = M \\ h_{21} = 1 \end{matrix} \quad \begin{matrix} \rightarrow T^2 S^3 \\ \text{deformed} \\ \text{conifold.} \end{matrix}$$

Mirror Symmetry:

Resolved Conifold \longleftrightarrow Deformed Conifold

(13)

Consider the 3 isometries

$$\left. \begin{aligned} x &\rightarrow x + C_1 \\ y &\rightarrow y + C_2 \\ z &\rightarrow z + C_3 \end{aligned} \right\} \text{natural } T^3 \text{ structure.}$$

Can now take

$$|dz_1|^2 = |dx - \tau_1 d\theta|^2$$

$$|dz_2|^2 = |dy - \tau_2 d\theta|^2$$

and change

$$\tau_1 \rightarrow \tau_1 + f_1$$

$$\tau_2 \rightarrow \tau_2 + f_2$$

$$|dz|^2 = 1 \rightarrow |d\tilde{z}|^2 = 1 - \epsilon$$

Then consider $f_i \rightarrow \infty$, $\epsilon \rightarrow 0$

with $f_1, f_2, \epsilon = \text{finite}$

(14)

Perform T-dualities in x, y, z

$$T_x: \begin{cases} G'_{\mu\nu} = G_{\mu\nu} - \frac{G_{x\mu} G_{x\nu} - B_{x\mu} B_{x\nu}}{G_{xx}} \\ B'_{\mu\nu} = B_{\mu\nu} + \frac{2 B_{x[\mu} G_{\nu]x}}{G_{xx}} \end{cases}$$

$$\Downarrow T_x \cdot T_y \cdot T_z$$

$$dS_{IIA}^2 = g_1 (dz + \Delta_1 \cot \theta_1 d\hat{x} + \Delta_2 \cot \theta_2 d\hat{y})^2 + g_2 (d\theta_1^2 + dx^2) + g_3 (d\theta_2^2 + dy^2) + g_4 [f_1 f_2 \epsilon d\theta_1 d\theta_2 - dx dy]$$

For $f_1 f_2 \epsilon = 1$, this is the metric of a "delocalized" deformed conifold.

Def. conifold has an extra term

$$\cos \Psi (d\theta_1 d\theta_2 - d\phi_1 d\phi_2 \sin \theta_1 \sin \theta_2) + \sin \Psi (d\phi_1 d\theta_2 \sin \theta_1 + d\phi_2 d\theta_1 \sin \theta_2)$$

①5 "Delocalize in Ψ " \Rightarrow $\Psi = \text{constant}$.

Dependence on Ψ of deformed conifold



Deformed Conifold is not toric and one needs to start from Resolved Conif.

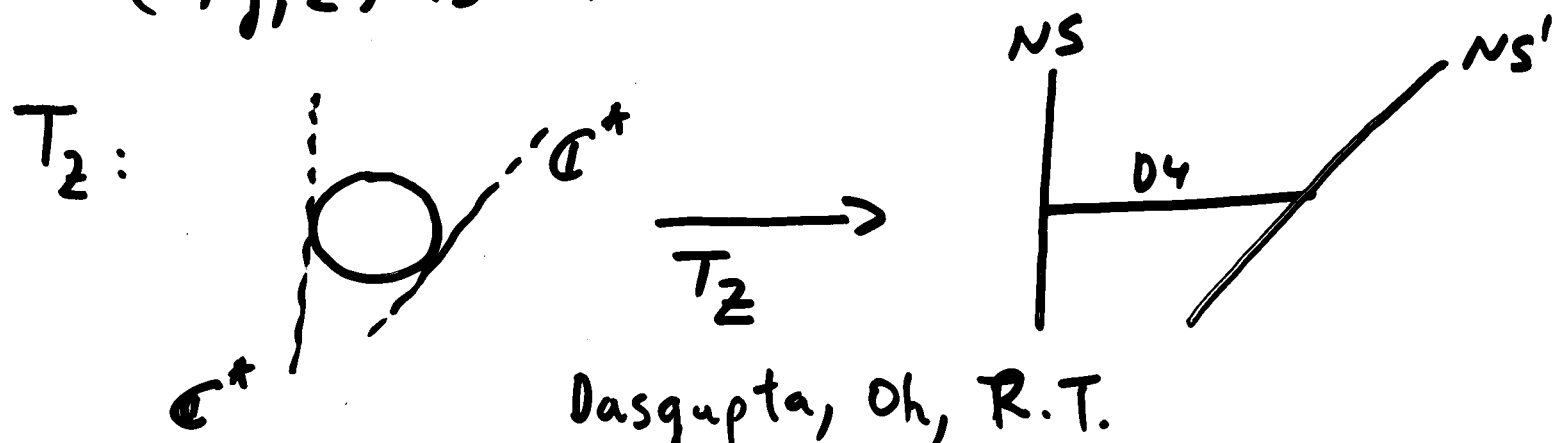
This can be seen also from following arguments:

a) Resolved Conifold

$$\begin{matrix} \mathcal{O}(-1) \oplus \mathcal{O}(-1) & (y \text{ is } S' \subset \mathbb{C}^*) \\ \downarrow & \\ \mathbb{P}^1 & (\mathbb{Z} \text{ - angular direction}) \end{matrix}$$

(x is $S' \subset \mathbb{C}^*$)

(x, y, z) is T^3



Dasgupta, Oh, R.T.
hep/th-0105166

(16)

b) Deformed Conifold

$$xy - uv = \mu$$

$$S_1: \begin{cases} x \rightarrow e^{i\theta} x \\ y \rightarrow e^{-i\theta} y \end{cases} ; S_2: \begin{cases} u \rightarrow e^{i\psi} u \\ v \rightarrow e^{-i\psi} v \end{cases}$$

But we cannot identify the third S'

\Rightarrow The mirror symmetry can be considered $\begin{matrix} \text{II B on res. conif.} \\ \Downarrow \\ \text{II A on def. conif.} \end{matrix} \left. \vphantom{\begin{matrix} \text{II B on res. conif.} \\ \Downarrow \\ \text{II A on def. conif.} \end{matrix}} \right\} \begin{matrix} \text{open} \\ \text{string} \\ \text{side} \end{matrix}$

As the "delocalization" will go through the transition, the mirror symmetry will be possible to discuss

for $\begin{matrix} \text{II A on res. conif.} \\ \Downarrow \\ \text{II B on def. conif.} \end{matrix} \left. \vphantom{\begin{matrix} \text{II A on res. conif.} \\ \Downarrow \\ \text{II B on def. conif.} \end{matrix}} \right\} \begin{matrix} \text{closed} \\ \text{string} \\ \text{side.} \end{matrix}$

(17)

Result:

$$dS^2 = g_1 [d\hat{z} + \Delta, \cot \hat{\theta}, d\hat{x} + \Delta_2 \cot \hat{\theta}, d\hat{y}]^2 \\ + g_2 [d\theta_1^2 + d\hat{x}^2] + g_3 [d\theta_2^2 + d\hat{y}^2] \\ + g_4 [d\theta_1 d\theta_2 - d\hat{x} d\hat{y}]$$

$$\left[\begin{aligned} d\hat{z} &= dz - b_{z\mu} dx^\mu = dz \\ d\hat{x} &= dx - b_{x\theta_1} d\theta_1 \\ d\hat{y} &= dy - b_{y\theta_2} d\theta_2 \end{aligned} \right. \quad (1)$$

This is exactly the metric for wrapped D6 on S^3 , but with redefinitions (1).

$$\left. \begin{aligned} db_{x\theta_1} &\neq 0 \\ db_{y\theta_2} &\neq 0 \end{aligned} \right] \text{ but small}$$

(18)

Our $\mathbb{II}A$ manifold has an almost complex structure J_i^k and the fundamental 2-form:

$$F_{ij} = J_i^k g_{kj}$$

Also, there is a (3,0) form:

$$\Omega_{ijk} \sim e_i \wedge e_j \wedge e_k$$

We observe that

$$\left. \begin{array}{l} dJ \neq 0 \\ d\Omega \neq 0 \end{array} \right\} \begin{array}{l} \text{Non-Kähler and} \\ \text{Non-complex manifold} \end{array}$$

We have thus described the first half of the

"modified geometric transition".

D

The modified type IIA transition:

D6 branes on a
non-Kähler deformation of a
deformed Conifold

↑ open - closed duality
↓

RR flux on \mathbb{P}^1
of a non-Kähler deformation of a
resolved Conifold

$$d\Omega \neq 0 \xleftrightarrow{\text{D.C.}} d\Omega \neq 0 \xleftrightarrow{\text{R.C.}}$$

Continuous way to understand this?
Use G_2 manifolds (7 dim. manifolds
preserving
 $N=1$ in 4d)

(19)

Observations

① Topological duality:
Chern-Simons on S^3 (inside non-Kähler)



Topological strings on
non-Kähler

Need to clarify the computation
of superpotential for almost complex
manifold

② Compute torsion classes

τ_1, \dots, τ_5

(20)

Usual story (Atiyah-Maldacena-Vafa)

II A : N D6 on S^3
(0123456) (456)

M theory

$\mathbb{R}^4/\mathbb{Z}_N \times S^3$
(789,10) (456)
Cone over $\tilde{S}^3/\mathbb{Z}_N \times S^3$

Flop $\tilde{S}^3 \leftrightarrow S^3$
 $\tilde{S}^3/\mathbb{Z}_N \times S^3 \rightarrow \tilde{S}^3 \times S^3/\mathbb{Z}_N$

II A

Hopf fiber
 \downarrow
 IP^1 with N units of RR flux

Resolved conifold with N units of RR flux on IP^1

1) (Brandhuber, Gomis, Gubser, Gukov)

$$\text{Symmetries: } x_1^2 + x_2^2 + x_3^2 + x_4^2 = r^2$$

$$\text{rotation } SO(4) \sim SU(2) \times SU(2)$$

$$\mathbb{Z}_2: \Sigma_i \leftrightarrow -\Sigma_i$$

$U(1)$: M theory circle

$\Rightarrow G_2$ holonomy metric has $SU(2)^2 \times U(1) \times \mathbb{Z}_2$

$$SU(2)_1: d\sigma_a = -\frac{1}{2} \epsilon_{abc} \sigma_b \wedge \sigma_c$$

$$SU(2)_2: d\Sigma_a = -\frac{1}{2} \epsilon_{abc} \Sigma_b \wedge \Sigma_c$$

$$U(1): \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}$$

$$\mathbb{Z}_2: \sigma_a \leftrightarrow \Sigma_a$$

Most general metric:

$$e^i = A(r) (\sigma_i - \Sigma_i), i=1,2; e^3 = D (\sigma_3 - \Sigma_3)$$

$$e^5 = B(r) (\sigma_2 + \Sigma_2); e^6 = C(r) (\sigma_3 + \Sigma_3)$$

$$e^7 = \frac{dr}{C(r)}; e^4 = B(r) (\sigma_1 + \Sigma_1)$$



What happens if one starts with D6 on a m-Kähler deformed conifold?

Need the 11-dimensional metric.

$$G_{\mu\nu}^M = e^{-\frac{2\phi}{3}} g_{\mu\nu}^{\bar{II}A} - e^{\frac{4\phi}{3}} A_\mu A_\nu$$

$$G_{M\bar{II}}^M = -e^{\frac{4\phi}{3}} A_\mu$$

A_μ - gauge field in $\bar{II}A$

To get $\bar{II}A$ gauge field, start with $\bar{II}B$ RR 3-form and 5-form

$$H^{(3)} = C_1 (dz \wedge d\theta_2 \wedge dy - dz \wedge d\theta_1 \wedge dx) +$$

$$+ C_2 \cot \hat{\theta}_1 dx \wedge d\theta_2 \wedge dy -$$

$$- C_3 \cot \hat{\theta}_2 dy \wedge d\theta_1 \wedge dx$$

$$F^{(5)} = k(r) (1 + *) dx \wedge dy \wedge dz \wedge d\theta_1 \wedge d\theta_2$$

Apply T-duality rules:

$$\tilde{F}_{ijk\dots}^{(n)} = F_{xijk}^{(n+1)} - n B_x [{}^i F_{jk\dots}^{(n-1)} + n(n-1) j_{xx}^{-1} \cdot B_x [{}^i j_{xj} F_x^{(n-1)}]]$$

$$\tilde{F}_{xijk\dots}^{(n)} = F_{ij\dots}^{(n-1)} - (n-1) j_{xx}^{-1} j_x [{}^i F_x^{(n-1)}]$$

We get:

$$H^{(3)}: F_{z\theta_1} = -\kappa_3 \cot \theta_2$$

$$F_{z\theta_2} = -\kappa_2 \cot \theta_1$$

$$F_{y\theta_1} = C_1 - 2\kappa_3 \propto B \cot \theta_2$$

$$F_{x\theta_2} = C_1 - 2C_2 \propto A \cot \theta_1$$

$$F_S: F_{\theta_1\theta_2} = k(r) - \kappa_1 (b_{x\theta_1} - b_{y\theta_2}) - 2C_3 \propto B b_{y\theta_2} \cot \theta_2 + 2C_2 \propto A b_{x\theta_1} \cot \theta_1$$

(24)

Therefore

$$A_{\mu} dx^{\mu} = \Delta_1 \cot \theta_1 d\hat{x} - \Delta_2 \cot \theta_2 d\hat{y}$$

Δ_i - functions of θ_i, x, y, z

M-theory metric:

$$dS^2 = e^{-\frac{2\phi}{3}} (h^{-1/2} dS_{0123}^2 + h^{1/2} \gamma' dr^2) +$$

$$+ e^{-\frac{2\phi}{3}} (dz + \Delta_1 \cot \theta_1 d\hat{x} + \Delta_2 \cot \theta_2 d\hat{y}) + e^{\frac{4\phi}{3}} (dx_{11} + \Delta_1 \cot \theta_1 dx - \Delta_2 \cot \theta_2 dy)^2$$

We can then identify

$$dz = d\psi_1 - d\psi_2$$

$$dx_{11} = d\psi_1 + d\psi_2$$

and we can write our metric as

$$\sigma_3 = d\psi_1 + \Delta_1 \cot \hat{\theta}_1 d\hat{x}$$

$$\Sigma_3 = d\psi_2 + \Delta_2 \cot \hat{\theta}_2 d\hat{y}$$

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$$ds_7^2 = \alpha_1^2 \sum_{a=1}^2 (\sigma_a + \Sigma_a)^2 + \alpha_2^2 \sum_{a=1}^2 (\sigma_a - \Sigma_a)^2$$

$$+ \alpha_3^2 (\sigma_3 + \Sigma_3)^2 + \alpha_4 (\sigma_3 - \Sigma_3)^2 +$$

$$+ \alpha_5^2 dr^2$$

$$\alpha_1 = \frac{1}{2} \sqrt{g_4 + 2g_2} \quad \alpha_3 = 1$$

$$\alpha_2 = \frac{1}{2} \sqrt{2g_2 - g_4} \quad \alpha_4 = \sqrt{g_1}$$

$$\alpha_5 = \sqrt{r' \sqrt{h}}$$

Torsion:

$$\tau = - * d\tilde{\Omega} - * \left[\frac{1}{3} * (* d\tilde{\Omega} \wedge \tilde{\Omega}) \wedge \tilde{\Omega} \right]$$

$$\tilde{\Omega} = \mathcal{F} \wedge e^7 + \Omega +$$

we have 4 torsion classes

τ_1	τ_2	τ_3	τ_4	
1	14	27	1	of $so(7)$

(26)

What about flop?

- we lifted the deformed conifold
on x_{11}

- we can now descend on \mathbb{Z}
and result is:

$$ds^2 = A [d\theta_1^2 + (dx - b_{x\theta_1} d\theta_1)^2] + \\ + B [d\theta_2^2 + (dy - b_{y\theta_2} d\theta_2)^2] + \\ + C [dx_{11} + \Delta_1 \cot \theta_1 (dx - b_{x\theta_1} d\theta_1) + \\ + \Delta_2 \cot \theta_2 (dy - b_{y\theta_2} d\theta_2)]^2$$

This is exactly the metric
of a non-Kähler resolved conifold.

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The final result is a non-Kähler deformation of a resolved conifold

$$dJ \neq 0$$

$$d\Omega \neq 0$$

Superpotential $\int (J + iB) \wedge d\Omega$

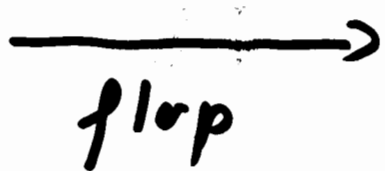
Torsion classes:

open $\bar{H}^1 A$
(w_1, \dots, w_5)

closed $\bar{H}^1 A$
(w'_1, \dots, w'_5)



G_2
(τ_1, \dots, τ_4)



G_2
(τ'_1, \dots, τ'_4)

it is necessary a consistency check.

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Conclusions

- Claims about non-Kählerity turn out true
- the specific example of mirror symmetry deformed conifold \leftrightarrow resolved conifold: detailed description of mirror symmetry and the flop transition
- Main lesson: nonKähler geometry is a necessary ingredient
- One should learn much more about them (classification, SUSY preserved, etc.)