Problem #1: The light from a star is measured in the V band. The star is observed to have a parallax angle of 0.04". You can assume that \( m_V \) is small and so the star appears very bright (lots of photon intensity).

a) Estimate the distance to the star (in meters).

\[
\frac{d}{\theta} = \frac{1}{p_c} \Rightarrow \quad p_c = \frac{1}{0.04} = 25 \text{pc} = 7.7 \times 10^7 \text{m}
\]

b) What is the minimum diameter telescope needed (in meters) to observe the parallax angle? Choose a reasonable observation wavelength in the V band.

\[
\theta = 1.22 \frac{\lambda}{D} \quad \text{Optrical Theorem}
\]

So \( D > 1.22 \frac{\lambda}{\theta} = ? \quad \lambda \text{ in middle of V band} = 550 \text{ nm} \)

\[
\theta = \frac{0.04 \pi}{(60)^2 (480)} \text{ rad} = 1.94 \times 10^{-7} \text{ rad}
\]

\[
D > 1.22 \frac{550 \times 10^{-9}}{1.94 \times 10^{-7}} = 3.46 \text{ meters}
\]

c) What focal length \( f \) is needed (in meters) for this telescope if you were to measure the star's parallax angle with a CCD camera which had 1 \( \mu \text{m} \) pixels?

\[
\frac{dy}{d\theta} = \frac{1}{f} \quad \text{so} \quad f = \frac{dy}{d\theta} = \frac{1 \times 10^{-6}}{1.94 \times 10^{-7}} = 5.15 \text{ meters}
\]
Problem #2: A star has temperature $T=8000 \, \text{K}$ and radius $R=0.5 \, R_\odot$.

a) What is the peak wavelength of emission for the star, and the peak wavelength of emission for the planet?

b) Calculate the absolute bolometric magnitude of the star.

\[ \lambda_{\text{max}} = \frac{0.00289}{\text{mK}} \]

\[ \lambda_{\text{max}} = \frac{0.00289}{8000} = 3.61 \times 10^{-7} \, \text{m} \]

\[ = 3.61 \, \mu \text{m} \]

In the blue $\rightarrow$ UV

\[ L = \left( \frac{4 \pi R^2}{L_\odot} \right) 6 T^4 \]

\[ = 4 \pi \left( 0.25 \, R_\odot \right)^2 \left( 5.67 \times 10^{-8} \right) \left( 8000 \right)^4 \]

\[ = 71 \left( 6.95 \times 10^{8} \right)^2 \left( 5.67 \times 10^{-8} \right) \left( 4.09 \times 10^{15} \right) \, \text{W} \]

\[ = 3.52 \times 10^{26} \, \text{W} = 0.93 \, L_\odot \]

\[ M = M_\odot - 2.5 \log_{10} \left( \frac{L}{L_\odot} \right) \]

\[ M = +4.74 - 2.5 \log(0.93) \]

\[ M = +4.82 \]
Problem #3: A star is observed to have a Ly$\alpha$ line $\lambda_{Ly\alpha} = 207.34$ nm

Note the usual Ly$\alpha$ line measured on Earth has $\lambda_{Ly\alpha} = 121.567$ nm.

a) Calculate the redshift $z$ of the star.
b) Calculate the radial velocity of the star, and indicate if the star is moving towards Earth or away from Earth.

$$z = \frac{\Delta \lambda}{\lambda_{rest}} = \frac{207.34 - 121.567}{121.567}$$

$$z = +0.705$$

b) This is redshift - moving away from Earth.

$$v = \sqrt{\frac{1 + \beta}{1 - \beta}}$$

$$\frac{(z+1)^2}{(z+1)^2 - 1} = \beta$$

$$\frac{(1 - \beta)(z+1)^2}{(z+1)^2 - 1} = 1 + \beta$$

$$\left(\frac{1.705}{1.705} - 1\right) = \beta\left(\frac{(z+1)^2}{(z+1)^2 + 1}\right)$$

$$\frac{(2+1)^2-1}{(2+1)^2+1} = \beta$$

$$\frac{(1.705)^2-1}{(1.705)^2+1} = \beta$$
Problem #1: Stars not only contain Hydrogen (charge Z=1, atomic Mass A = 1) but also contain the isotope Deuterium (charge Z=1, atomic mass A = 2). Deuterium is similar to hydrogen in every respect, except that there is a neutron in the nucleus, in addition to the proton. This extra neutron will modify the energy spectrum slightly.

a) Calculate the reduced mass of the Deuterium atom.

\[ m = \frac{m_e m_D}{m_e + m_D} = \frac{m_e \sqrt{m_p + m_n}}{m_e + m_p + m_n} \]

\[ = \frac{9.1 \times 10^{-31} \left( 1.67 \times 10^{-27} + 1.67 \times 10^{-27} \right)}{9.1 \times 10^{-31} + 1.67 \times 10^{-27} + 1.67 \times 10^{-27}} \]

\[ = 9.1 \times 10^{-31} \left( 3.345 \times 10^{-27} \right) \]

\[ = \frac{9.1 \times 10^{-31}}{3.345 \times 10^{-27}} \]

\[ M_D = 0.999727 \, m_e \]

For Hydrogen \( M_H = 0.99945 \, m_e \)

\[ \frac{M_D}{M_H} = 1.000277 \]

b) Using the Bohr model, calculate the wavelength of the Dα Balmer line (transition from n=3 to n=2).

Compute the wavelength difference \( \Delta \lambda = \lambda_{\text{He}} - \lambda_{\text{Dα}} \).

c) If I was to use a diffraction grating to observe this splitting, and we are looking at the 2nd order of the interference pattern, estimate the minimum number of lines needed in the grating to be able to observe the Dα line in a star which also has the Hα line.

\[ E_N = -\frac{M_D e^4}{3 \pi \hbar^2 \epsilon_0^2 n^2} \]

\[ \rightarrow -\frac{M_D e^4}{3 \pi \hbar^2 \epsilon_0^2 k^2 n^2} \]
Balmer $H\alpha$ line $\lambda = 656.281$ nm

So $\text{Debye energy} \ E = h\nu = \frac{hc}{\lambda}$

$\lambda_D = \frac{hc}{E_D}$

$\Delta \lambda = \lambda_D - \lambda_H = \frac{hc}{E_D} - \frac{hc}{E_{H\alpha}}$

$= \frac{hc}{1,000277E_{H\alpha}} - \frac{hc}{E_{H\alpha}}$

$= \left( \frac{1}{1,000277} - 1 \right) \frac{hc}{E_{H\alpha}}$

$\Delta \lambda = \left( \frac{1 - 1,000277}{1,000277} \right) \frac{hc}{E_{H\alpha}} = 7.000277 \times \lambda_{H\alpha}$

$= 0.182$ nm

(6) $\Delta \lambda = \frac{2}{\eta N} = 0.182$ nm

$N = \frac{\lambda}{\eta \times (0.182 \times 10^{-9})} = \frac{656.281 \times 10^{-9}}{(2)(0.182 \times 10^{-9})} = 1803$

So need grating with at least $\geq 1800$ lines
Problem #2: A star has temperature $T=8000$ K and radius $R_s=0.5\ R_\odot$. There is a planet orbiting the star with semi-major axis $a=4.5\ \text{AU}$, and eccentricity $e=0$. The planet has a radius $R_p=2.3\ R_j$ where $R_j$ is Jupiter's radius.

a) Estimate the Temperature of the planet. You can assume both the star and the planet is emitting like a blackbody radiator. You also should assume the only source of energy for the planet is the star, and all light falling onto the planet is absorbed.

b) What is the peak wavelength of emission for the star, and the peak wavelength of emission for the planet?

c) Calculate the difference in absolute bolometric magnitudes between the star and the planet.

\[
L = \frac{4\pi R^2}{6} T^4 = \frac{4\pi (0.25 R_\odot)^2}{6} (8000)^4
\]

\[
= \frac{\pi}{6} \left(6.95 \times 10^8\right)^2 (5.67 \times 10^{11})(4.96 \times 10^{15})
\]

\[
= 3.52 \times 10^{26} \text{W} = 0.93 \ L_\odot
\]

planet: gets Flux $\frac{L_\odot}{4\pi a^2}$ at distance $a$ from planet

\[
\text{Flux} = \frac{0.93 L_\odot}{4\pi (4.5)^2 (1.49 \times 10^9)^2} = 1.64 \times 10^{-25} L_\odot/\text{m}^2
\]

\[
\text{Area of planet getting sun} = \pi R_p^2
\]

\[
A_p = \pi (2.3)^2 (1.2)^2 (6.37 \times 10^6)^2
\]

\[
= 8.46 \times 10^{16} \text{m}^2
\]

Energy in is $(\text{Flux})(\text{Area}) = (1.64 \times 10^{-25})(8.46 \times 10^{16}) = 1.38 \times 10^{-8} \ L_\odot
\]

\[
= 5.31 \times 10^{18} \text{W}
\]

Now I assume planet in all at same temperature.

Output luminosity = input energy $= 5.31 \times 10^{18} \text{W} = \frac{4\pi R_p^2}{6} T_p^4$
\[ T_P^4 = \frac{5.31 \times 10^{18}}{4 \pi R_P^2} = \frac{7 R_P^2 (1.64 \times 10^{-25}) L_0}{4 \pi R_P^2} \]

\[ = \frac{1}{46} (1.64 \times 10^{-25}) L_0 \]

\[ L_0 = 7.23 \times 10^{-19} L_\odot \]

\[ T_P = 2.77 \times 10^8 \]

\[ T = 129^\circ K \quad \text{[Cold, !]} \]

\( \Box \)

\[ \lambda_{\text{max}} T = 0.002847 \text{ m K} \]

\[ \lambda_{\text{max star}} = \frac{0.002847}{8000} = 3.61 \times 10^{-7} \text{ m} \]

\[ = 3.61 \text{ nm} \]

\[ \lambda_{\text{max planet}} = \frac{0.002847}{129} \text{ m} = 2.2 \times 10^{-5} \text{ m} \]

\[ = 2242 \text{ nm} \]

\[ \text{In the IR.} \]

\( \Box \)

\[ M = M_{\text{star}} - 2.5 \log_{10} \left( \frac{A_{\lambda}}{M_\odot} \right) \]

\[ M_{\text{star}} = +4.74 - 2.5 \log_{10} (0.93) = +4.74 + 0.0788 = +4.82 \]

\[ M_{\text{planet}} = +4.74 - 2.5 \log_{10} (1.38 \times 10^{-8}) = 4.74 + 19.65 = +24.39 \quad \text{Very dim.} \]

\[ M_{\text{star}} - M_{\text{planet}} = m_{\text{star}} - m_{\text{planet}} = -19.57 \]
Textbook ok, printout of online relativity notes ok, calculators ok. No other notes allowed.

Problem #3: A star is observed to have a split Ly$\alpha$ lines, the observed Ly$\alpha$ line has three wavelengths:

\[
\lambda_{\text{Ly} \alpha}^- = 307.3533 \, \text{nm} \\
\lambda_{\text{Ly} \alpha 0} = 307.3568 \, \text{nm} \\
\lambda_{\text{Ly} \alpha}^+ = 307.3603 \, \text{nm}
\]

Note the usual Ly$\alpha$ line measured on earth has $\lambda_{\text{Ly} \alpha} = 121.567 \, \text{nm}$.

Interpret the shifting of the spectral lines as being a combination of the Zeeman effect and the radial motion of the star.

a) Calculate the redshift $z$ of the star.
b) Calculate the radial velocity of the star, and indicate if the star is moving towards Earth or away from Earth.
c) Calculate the strength of the magnetic field in the star.

\[ d \text{ Centauri line is } m_e = 0 \text{ state } \Rightarrow \text{ ZEEMAN SHIFT } \]

\[ z = \frac{\Delta \lambda}{\lambda_{\text{rest}}} = \frac{307.3568 - 121.567}{121.567} = \frac{185.79}{121.567} = 1.528 \]

b) This is a red shift so the star is moving radially away from earth.

\[ z + 1 = \sqrt{\frac{1 + \beta}{1 - \beta}} \]

\[ (z + 1)^2 = \frac{1 + \beta}{1 - \beta} \quad (1 - \beta)(z + 1)^2 = 1 + \beta \]

\[ (z + 1)^2 - 1 = \beta (1 + (z + 1)^2) \]

\[ \beta = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1} = \frac{1.528^2 - 1}{1.528^2 + 1} = \frac{1.539}{3.139} = 0.739 \]

So \( v_T = 0.84 \, \text{c} \)

\[ v = v_0, \quad v_0 \pm \frac{e \beta}{4 \pi \mu} \]

No hint needed to calculate constant of splitting. \( v_0, \quad v_c \pm \frac{2}{\gamma \mu} \)
\[ \lambda_0 = 121.567 \text{ nm} \quad \text{note you need to calculate in atom's rest frame!} \]

\[ \nu_0 = \frac{c}{\lambda} = \frac{3 \times 10^8}{121.567} = 2.46 \times 10^8 \text{ Hz} \]

Red shift changed \( \lambda \) by factor \( 1 + z \)

\[ \lambda_{\text{ste}2} = (1 + z) \lambda_{\text{rest}} \]

\[ \nu_+ = \frac{c}{\lambda_{\text{rest}}} = \frac{c}{(\lambda_{\text{ste}2})/(1+z)} \]

\[ \nu_+ = \frac{(1+z)c}{\lambda_{\text{ste}2}} = \nu_0 + \frac{eB}{4\pi M} \]

\[ \nu_{\text{rest}} = \frac{(1+z)c}{\lambda_{\text{ste}2}} = \nu_0 - \frac{eB}{4\pi M} \]

Substituting:

\[ \frac{(1+z)c}{\lambda_{\text{ste}2}} = \left( \frac{1}{\lambda_{\text{ste}2} - \lambda_{\text{ste}2+}} \right) = \frac{2eB}{4\pi M} \]

\[ B = \frac{2\pi M (1+z)c}{\lambda_{\text{ste}2} - \lambda_{\text{ste}2+}} \]

\[ \sim M_e \quad c \left( \lambda_{\text{ste}2} - \lambda_{\text{ste}2+} \right) \]

\[ = \frac{2\pi (9.1 \times 10^{-31}) (2.528)(3 \times 10^8)}{e} \left( \begin{array}{c} -10^9 \ \ + \ 10^9 \\ \frac{307.3603}{307.3533} \end{array} \right) \]

\[ = \frac{4.33 \times 10^{-12}}{e} \left( 7.41 \times 10^{-8} \right) = \frac{3.20 \times 10^{-14}}{1.60 \times 10^{-14}} = 2 \text{ Tesla}. \]