Homework IV

due September 29nd 2006

Very high energy gamma-rays from distant galaxies are absorbed by the intergalactic electromagnetic radiation fields. Two photons γ_1 and γ_2 can interact and produce an electron e^- and an anti-electron (or positron) e^+ : $\gamma_1+\gamma_2\rightarrow e^-+e^+$. A positron has the same mass as an electron and an opposite electric charge. The mass of an electron is $0.511\,MeV/c^2$. In this exercise, we consider a gamma-ray γ_1 of energy $E_1=1\,TeV$ propagating through the inter-galactic environment until it interacts with a gamma ray γ_2 of energy E_2 . These energies are expressed in the Earth reference frame. The two photons propagate in opposite directions and collide head-on.

1) Write the energy momentum for the system before the interaction.
2) What is the velocity β of the system rest frame? Also express the Lorentz factor γ as well as the product $\beta\gamma$.
3) Applying a Lorentz transform, write the energy of the system in its rest frame.
4) What is the smallest energy E_2 of γ_2 for the electron-positron pair production to still be possible?
5) By using the system invariant mass, re-establish the expression for the smallest energy E_2 that can still permit the reaction.
6) Calculate the largest value of λ_2 , the wave-length of γ_2 , for the pair creation reaction to still be possible when $E_1=1\,TeV$ ($h=6.26\times10^{-34}J\cdot s$, $c=3\times10^8 m\cdot s^{-1}$ and $1\,eV=1.6\times10^{-19}J$)