Homework VII

due October 27^h 2006

Alpha decay of Uranium 238

Uranium 238 ($M_{^{238}U}=238.050784\,u$) decays in Thorium 234 ($M_{^{234}Th}=234.043593\,u$) by emitting an α particle of mass $M_{\alpha}=4.002602\,u$. Uranium 238 has a life time $\Delta t=4.4\times10^{9}$ years. The electric charge of an Uranium nucleus is $+92\,e$ while the electric charge of an α particle is $+2\,e$ with $e=1.602\times10^{-19}\,C$.

1) What is the kinetic energy K_{α} of the emitted α particle (you can neglect the recoil, $1\,u=931.49401\,MeVc^{-2}$)
2) What is the uncertainty ΔK_{α} of quantum origin on the energy K_{α} ?
3) What is the momentum of the α particle? Do you need to use relativistic relations to calculate this momentum ($1\,u=10^{-3}\,kg/N_A$ with $N_A=6.022142\times10^{23}$)? *4e3
4) Assuming the α particle is confined in the volume of the nucleus until the decay when it escapes, estimate the size of the nucleus (consider the nucleus to be one-dimensional).
5) How much time does it take for the α particle to travel across the nucleus?
6) In the classical picture where the α particle is bouncing off the edge of the nucleus until it escapes, how many times does it typically travel across the nucleus before the decay occurs.? The α particle can be considered to have some chance P_{Escape} of escaping each time it approaches the edge of the nucleus. Estimate P_{Escape} .
7) Draw a graph of the potential energy for the interaction between the α particle and the rest of the nucleus as a function of the distance from the center of the nucleus. Assume, as we implicitly did, the potential is zero inside the nucleus. Draw a horizontal line across your graph to represent the kinetic energy of the α particle.
8) In order for the α particle to escape the nucleus it must violate energy conservation for a short time interval Δt . Estimate Δt from your estimate of the nucleus size and from the repulsive Coulomb potential between the α particle and the rest of the nucleus (hint: estimate the Coulomb potential at $\sim1\,fm$ distance, $1/4\pi\epsilon_0=9.987\times10^{9}\,N\cdot m^{2}\cdot C^{-2}$).
9) How fast must the α particle be traveling when it traverses the Coulomb potential barrier? This result should surprise you. Make a few comments. Can you identify approximations we did that could be changed so we obtain a more acceptable result?