# Homework VIII

*due November 3h 2006*

**Semi-infinite square well**

We consider a particle of mass $m$ in a potential $V(x)$ defined in three regions along the $x$ axis as follows:

Region I $x \le 0$, $v(x) = \infty$
Region II $0 < x < a$, $v(x) = -V_0$
Region III $x > a$, $v(x) = 0$

1) Write the particle wave function $\psi(x)$ in region I
2) What is the range of possible energies for bound states?
3) Write Schrodinger equation in region II. In order to lighten the notation, introduce $k = \sqrt{2m(E+V_0)/\hbar^2}$
4) Write the general form of the solutions to Schrodinger's equation in region II as a linear combination of sin and cos functions. How is this general solution affected by the boundary condition in $x = 0$ ?
5) Write the Schrodinger equation in region III. In order to lighten the notation, introduce $\alpha = \sqrt{-2mE/\hbar^2}$
6) Write the general form of the solutions to Schrodinger's equation in region III as a linear combination of two exponentials. How is this solution affected by the normalization condition?
7) Write the equations resulting from the boundary conditions in $x = a$
8) Using the boundary conditions equations and the expressions for $k$ and $\alpha$ , obtain the energy quantization equation. Introducing $\theta = k \cdot a = \sqrt{2ma^2(E+V_0)/\hbar^2}$ and $\theta_0 = \sqrt{2ma^2 V_0/\hbar^2}$ , put this equation in a simpler form.
9) Solve the energy quantization equation graphically for $\theta_0 = 1$ and $\theta_0 = 5/2$ . How many solutions are there in each case? What are the values of $\theta$ (round up to the closest integer)?
10) a) Express $\psi_{II}(x)$ and $\psi_{III}(x)$ in terms of $\theta$ , $\theta_0$ , $a$ , $x$ and your normalization constants.
    b) For the fundamental state with $\theta_0 = 5/2$ where is the particle the most likely to be found?
    c) Graph $\psi(x)$ .
    d) From the boundary conditions, obtain the relative normalization of $\psi_{II}(x)$ and $\psi_{III}(x)$ .
    e) Write the normalization condition for $\psi(x)$ . You do not have to calculate the normalization constant. If you had to, would that be difficult?
11) How different is the the fundamental state for $\theta_0 = 5/2$ from the first excited state of a finite square well of width $L = 2a$ ?