

Homework X

due November 24th 2006

1) Two dimensional square well

Consider a particle in a two dimensional infinite square well with sides a and $b = a/2$.

a) Find the lowest six energy levels, with their quantum numbers and degeneracy. (

$$E_{n_x, n_y} = \frac{\hbar^2 \pi^2}{2M} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} \right)$$

b) Write the time independent wave function for given n_x and n_y .

c) For $n_x = 2$ and $n_y = 3$, what are the points where the particle is the most likely to be found?

2) Hydrogen states degeneracy

Prove the degeneracy of the n^{th} level in the hydrogen atom is $2n^2$ (remember that $0 \leq l < n$ and $|m| \leq l$ and $m_s = \pm 1/2$) You will need to know that

$$\sum_{l=1}^n l = n(n+1)/2$$

3) Radial component in the 1s Hydrogen wave function

The radial equation for $l=0$ states in hydrogen is

$$\frac{d^2}{dr^2}(r \cdot R(r)) = \left(\frac{1}{n^2 a_B^2} - \frac{2}{r \cdot a_B} \right) (r \cdot R(r))$$

a) Verify that $R_{1s}(r) = e^{-r/a_B}$ is a solution

b) Calculate the probability density $P(r)$ of finding the electron at a given distance r from the nucleus in the $1s$ orbital. What is the distance r_0 where this probability density is maximal?

4) Zeeman effect

Consider two levels of the helium atom in both of which spins are anti-parallel and one electron is in an s state. In the higher level the other electron occupies a d state and in the lower level it occupies a p state. A magnetic field is applied along the z direction.

a) Sketch the splitting of both levels

b) Imagine all transitions from anyone of the d states with $L_z = m_l \hbar$ to anyone of the f states with $L_z = m_f \hbar$ are possible (15 possible transitions). How many different transition energies would there be?

c) In fact the only possible transitions are such that $|m_f - m_l| \leq 1$. How many transition energies are there?

5) **Pauli exclusion**

Consider 11 electrons in a two dimensional infinite square well with $a=b$. We will

suppose the electrons do not interact with each other and $E_{n_x, n_y} = \frac{\hbar^2 \pi^2}{2M} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} \right)$

- What is the ground state energy?
- What is the degeneracy of the ground state?
- What is the lowest energy transition from the ground state?

6) **Multi-electron atoms**

The electronic structure of Li ($Z=3$) is $1s^2 2s^1$.

- What is the electronic structures of Si ($Z=14$)?
- What is the electronic structure of Fe ($Z=26$)?
- What is the electronic structure of Rb ($Z=37$)?