Introduction to Modern Physics
PHYS3740
Midterm test 2

Friday November 11th 2006

11:50AM in JFB B1

Note: the best possible score is 175/100

1) **Infinite square well** (*note: all the questions are independent from each other*)
   A particle of mass m in an infinite one dimensional square potential of width a can be in quantum states of principal quantum number n with quantized energy
   $E_n = n^2 \frac{\pi^2 \hbar^2}{2\mathrm{m}\, a^2} = n^2 E_1$.
   a) *(10points)* A proton ($m_p = 1.673\times10^{-27} kg$) is in an infinite square well of width $a = 5\mathrm{fm}$. Calculate the energy of the ground state in Joules and in eV .
   ($1\mathrm{eV} = 1.602\times10^{-19} J$, $h = 6.63\times10^{-34} J\cdot s$).
   b) *(10points)* Assuming transitions from state n to state m are possible for all values of n and m , calculate the five lowest transition energies in units of E_1 . For each, indicate the states between which the transition takes place. Draw the corresponding energy line spectrum.
   c) *(10points)* Draw representations of the real (not complex) wave functions of the ground state and first 2 excited states of an infinite square potential. For the same states, draw a representation the probability density.
   d) *(10points) D*raw the representation of the real (not complex) wave function for the 3rd excited state of a particle in a finite square well of depth V_0 and width a . Is the energy of that state smaller or larger that the energy of the 3rd excite state in an infinite square well of width a ?
   e) *(10points)* A ping-pong table is $a = 3\mathrm{m}$ long. During a friendly game, the $m = 2\times10^{-3} kg$ ping-pong ball goes from one end of the table to the other in half a second. As long as the players do not fail sending the ball back, the ping-pong ball can be considered to be trapped in an infinite square well. Estimate n the principal quantum number of the quantum state the ping-pong ball is in (Hint: $E = \frac{1}{2} m v^2$).

2) **Arbitrary potential well**
   The $m = 2\times10^{-3} kg$ ping pong ball is bouncing on a hard surface. The vertical position of the ball is measured as x . The potential energy of the ball is $U(x) = \infty$ for $x \leq 0$ and $U(x) = mgx$ for $x > 0$ with $g = 10\mathrm{m}\cdot s^{-2}$.
   a) *(20 points)* Using Heisenberg uncertainty relation, estimate the energy of the quantum ground state for the bouncing ping-pong ball ($h = 6.63\times10^{-34} J\cdot s$).
   b) *(20 points)* In the ground state, how "high" would the ball bounce? (Hint: $E = mgH$)
   c) *(10 points)* How would your results change in the case of an electron whose mass is 2.2×10^{27} times smaller?

3) **Scientists Announce Creation of Atomic Element, the Heaviest Yet**

By Rick Weiss, Washington Post Staff Writer, Tuesday, October 17, 2006; Page A03

"Scientists in California and Russia announced yesterday that they have created the heaviest atomic element ever made, adding a new item to the universal menu of matter known as the periodic table and revealing fresh secrets about the nature of atoms, the fundamental units of physical stuff. (...) The three atoms of ununoctium created in the latest experiments came and went in a literal flash. But during their brief tenures of about nine ten-thousandths of a second ($9\times10^{-4}\,s$) each in a laboratory on Russia's Volga River, those three atoms revealed much about the laws that govern the behavior of matter, scientists said."

(25 points) How well can the mass of this new atom be known? ($h=6.63\times10^{-34}\,J\cdot s$ and $c=299792458\,\mathrm{m}\cdot s^{-1}$) Give your answer in kg and in eV/c^2

4) **Down the potential step** (*note: questions a,b and c are independent from each other*)

We consider a potential energy $V(x)=0$ for $x\leq0$ (region I) and $V(x)=-V_0$ for $x>0$ (region II). For states of energy $E>0$ the Schrodinger equation for particles of mass m reads $\frac{d^2\psi}{dx^2}=-k_I^2\psi$ in region I and $\frac{d^2\psi}{dx^2}=-k_{II}^2\psi$ in region II with $k_I=\sqrt{2\mathrm{mE}/\hbar^2}$ and $k_{II}=\sqrt{2\mathrm{m}(E+V_0)/\hbar^2}$. The general form of the solution is $\psi(x)=Ae^{ik_Ix}+Be^{-ik_Ix}$ in region I and $\psi(x)=Ce^{ik_Ix}+De^{-ik_Ix}$ in region II. The beam of particle is shot from region I toward region II.

a) *(10 points)* Explain why we can set $D=0$?

b) *(10 points)* Write the boundary condition equations in $x=0$.

c) *(10 points)* Draw the representation of a real (not complex) wave passing the potential step. Do we have bound states?

d) *(10 points)* The wave of amplitude A corresponds to the beam of particles shot toward the potential step. The wave of amplitude B corresponds to the particles reflected by the potential step. The wave of amplitude C corresponds to the particles transmitted through the potential step. Express B and C in terms of A , k_I and k_{II}

e) *(10 points)* A wave of amplitude A and wave vector k_I corresponds to a beam intensity (particles per seconds) $I_A=\hbar k_I A^2/m$. In the same way $I_B=\hbar k_I B^2/m$ and $I_C=\hbar k_{II} C^2/m$. Express the reflection and transmission coefficients R and T . Verify they add up to 1.