[1] Charge to mass ratio
A particle with a mass \( m \) and an electric charge \( q \) goes undeflected when traveling throughout uniform electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \) perpendicular to each other and both perpendicular to the particle’s trajectory. When the electric field is turned off, the particle is observed to follow a circular trajectory of radius \( R \). Express the charge to mass ratio \( q/m \).

\[
\text{The fact the particle goes undeflected when both } E \text{ and } B \text{ are present gives us } \frac{q}{m} = \frac{q}{\sigma B}.
\]

Then, the \( E \) field is turned off \( q/\sigma B = m/\sigma R \) so \( q/m = \frac{e}{\sigma R} = \frac{E}{\sigma R} \) \( \frac{q}{m} = \frac{E}{\sigma R} \).

[2] Compared strength of gravitational and electromagnetic interactions
Calculate the ratio between the electrostatic and gravitational force between two electrons.

The gravitational force between two electrons separated by a distance \( r \) is \( F_g = \frac{G m_1 m_2}{r^2} \) while the electric force is \( F_e = \frac{e^2}{4\pi \varepsilon_0 r^2} \). Using \( e = 1.6 \times 10^{-19} \text{ C}, \varepsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}, G = 6.7 \times 10^{-8} \text{ m kg}^{-1} \text{ s}^{-2} \) and \( m = 9.1 \times 10^{-31} \text{ kg} \)

we obtain \( \frac{F_e}{F_g} = 4.1 \times 10^{43} \).

[3] The mass of Yukawa’s meson
Considering nucleons to be separated by \( \delta r \approx 10^{-15} \text{ m} \) in the nucleus and considering the intermediary meson to be traveling between them at the speed of light, give an estimate of the mass of the Yukawa meson. \( \text{(Note this is not very meaningful. It is nothing more than an estimate based on dimensionality.)} \)

The time scale of the meson exchange between nucleons is \( \delta t = \frac{\delta r}{c} \). The mass must be within the range of the time and energy uncertainty relation \( \delta E \cdot \delta t \geq \frac{\hbar}{2} \). Using the equality limit and the relation \( \delta E = m c^2 \) we obtain

\[
m \geq \frac{\hbar c}{2 \delta r} = \frac{\hbar c}{2 \delta r} \approx \frac{2 \times 10^{-34} \text{ J s}}{2 \times 10^{-34} \text{ m} \cdot 10^{-16} \text{ m}} = 0.46 \times 10^{-23} \text{ kg}.
\]

More interestingly, we may calculate \( m c^2 = \frac{2 \times 10^{-34} \text{ J s} \cdot 3 \times 10^9 \text{ m s}^{-1} \cdot 3 \times 10^8 \text{ m} \text{ s}^{-1}}{1.6 \times 10^{19} \text{ J}} = 94 \text{ MeV} \)

\[ m c^2 = 94 \text{ MeV} \]
Nuclear electron model

Before the neutron was discovered, it was thought that electrons might be trapped inside the nucleus, electrically neutralizing an excess of protons. Considering the size of the nucleus to be \(6r \approx 10^{-15} \text{m}\) and using the Heisenberg uncertainty relation, estimate the energy of such electrons trapped in the volume of the nucleus. Since the electron is very light \((m_e c^2 = 511 \text{keV})\), it is easily relativistic so the relation between momentum \(p\) and energy \(E\) is \(E^2 = p^2c^2 + m_e^2c^4\). How is your result compared to the typical energy of the electron emitted in \(\beta\) decay, which is 1MeV?

From the momentum-position uncertainty relation \(\Delta p \Delta x > \frac{\hbar}{2}\), we obtain the order of the momentum of a nuclear electron as \(p \approx \frac{\hbar}{2\Delta x}\) and the energy \(E \approx \sqrt{\left(\frac{\hbar c}{2\Delta x}\right)^2 + (m_e c^2)^2}\). It is convenient to use \(\frac{\hbar c}{2\Delta x} \approx 137 \text{ keV fm}\) and \(m_e c^2 \approx 0.511 \text{ MeV}\) so the second term is negligible and \(E \approx 100 \text{ MeV}\) which is much larger than the typical \(\beta\) decay electron.

Building hadrons

(a) Using up to \(n\) quark flavors, how many different quark combinations can be used to build mesons?

A meson is a system of a quark and an antiquark bound together. There are \(n\) mesons built from a quark bound to its own antiquark like \(u\bar{u}\) or \(d\bar{d}\). There are \(n(n-1)\) mesons made from quarks of different flavors. So in total, with \(n\) flavors we can build \(N_n = n(n-1)\) mesons counting particles and antiparticles. This is only as far as quark composition goes.

(b) Using up to \(n\) quark flavors, how many different quark combinations can be used to build baryons?

Baryons are systems of three quarks bound together. There are \(n\) baryons made of three quarks of the same flavor.

There are \(n(n-1)(n-2)\) baryons with all quarks of one flavor. There are \(\frac{n(n-1)(n-2)}{3!}\) baryons made of three quarks of all different flavors. The denominator \(3! = 6\) is the number of ways to order 3 distinguishable objects.

In total we have \(N_b(n) = n^3 + \frac{n(n-1)(n-2)}{3} = \frac{1}{6} n^3(3n^2 - 3n + 2)\) which may be doubled to count antiparticles: \(N_b(n) = \frac{1}{2} n^3(3n^2 - 3n + 2)\).

(c) List all the possible combinations of the quarks \(u, d, s\) and \(c\) to build a meson. How many combinations are resulting in a charmed of +1, +2 and +3?

The 4 mesons with no flavor are \(u\bar{u}, d\bar{d}, s\bar{s}\) and \(c\bar{c}\). They have no net charm.

Mesons that are not flavor like but which have no charm are \(u\bar{d}, u\bar{s}, d\bar{s}, d\bar{c}\) and their antiparticles. There are 6 of them.

Mesons with a charm of +1 are \(c\bar{u}, c\bar{d}, c\bar{s}\) and \(c\bar{c}\). There are 3 of them. Their antiparticles have a charm of -1.

Thus we have 3 of them as well so we reach a total of 16 mesons, 3 of which have a charm of +1.

Because of their structure, mesons cannot not have any flavor change whose absolute value exceeds 1.
(d) List all the possible combinations of the quarks $u, d, s$ and $c$ to build a baryon. How many combinations are resulting in a charm of $+1$, $+2$ and $+3$?

Bayons with no charm are: $uuu, uud, udu, udd, uss, sdd,dds,sss$, and $sss$. There are 10 of them.

Bayons with a charm of $+1$ are: $ceu, cud, cus, cdd, cds$, and $css$. There are 6 of them.

Bayons with a charm of $+2$ are: $ccu, ccd$, and $ccs$. There are 3 of them.

There is only one baryon of charm $+3$, that is $ccc$.

<table>
<thead>
<tr>
<th>Charm</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of baryons</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

We recover the expected number of 30 baryons constructed from 4 quark flavors to be multiplied by 2 to count antibaryons.

[6] Composite model

In the 80’s, composite models were popular. They were based on the idea that elementary particle could in fact be themselves be composed of even more elementary particles. One such model is the rishon model due to H. Harari, (Physics Letter B 86(1): 83-86, 1979) and independently to M. Shupe (Physics Letters, 86B, 87, 1979). The model is based on two hypothetical elementary particles called rishons (primary in Hebrew). The rishons would be $T$ with an electric charge of $\frac{1}{3}$ and $V$, which would be neutral.

(a) Leptons and quarks in the first generation would be ordered triplets of rishons ($T$ and $V$) or anti-rishons ($\bar{T}$ and $\bar{V}$). In this model, the other generations would correspond to excited states of the particles in the first generation. Construct all leptons $e^-$, $\nu_e$, $e^+$, $\bar{\nu}_e$ and quarks $u$, $d$, $\bar{u}$, and $\bar{d}$, in the first generation. (Note that for the quarks, the number of permutations corresponds to the number of color charges)

$$e^- = \{\bar{T}, \bar{T}, \bar{T}\}$$ and correspondingly $e^+ = \{T, T, T\}$.

Similarly $\nu_e = \{\bar{V}, \bar{V}, \bar{V}\}$ and $\bar{\nu}_e = \{V, V, V\}$

In the same way $u = \{T, T, V\}$ and $\bar{u} = \{\bar{T}, \bar{T}, \bar{V}\}$

and $d = \{\bar{T}, \bar{T}, \bar{V}\}$ and $\bar{d} = \{T, T, V\}$

(b) In this model, intermediate bosons could be obtained from one set of three rishons and one set of three anti-rishons. Construct the $W^+$, $W^-$ and the $Z^0$ in a this way.

We could have $W^+ = \{T, T, T, \bar{V}, \bar{V}, \bar{V}\}$ and $W^- = \{V, V, V, \bar{T}, \bar{T}, \bar{T}\}$

while $Z^0 = \{V, V, V, \bar{V}, \bar{V}, \bar{V}\}$ or $Z^0 = \{\bar{T}, \bar{T}, \bar{T}, T, T, T\}$.

(c) Gluons could be constructed in the same way. How many such rishon-built gluons can you find? What restriction may you apply to get only 8 gluons?

Gluons could be constructed from triplet and triplet pairs with the same number of $T$ & $V$ to ensure neutrality. Triplet of identical rishons have already been used so we have $\{(T, T, V), (\bar{T}, \bar{T}, \bar{V})\}$ and $\{(v, v, v), (\bar{v}, \bar{v}, \bar{v})\}$ and all their triplet permutations amounting to $2 \times 3^3 = 54$. If we require additionally that two rishons, with the same flavor be always adjacent within an ordered triplet then we have $2 \times 2 \times 3^2 = 36$ possible combinations: $\{(T, T, V), (\bar{T}, \bar{T}, \bar{V})\}$, $\{(T, V, V), (\bar{T}, \bar{V}, \bar{V})\}$, $\{(V, T, V), (\bar{V}, \bar{T}, \bar{V})\}$, $\{(V, V, V), (\bar{V}, \bar{V}, \bar{V})\}$, $\{(\bar{T}, T, \bar{V}), (\bar{V}, \bar{T}, T)\}$, and $\{(\bar{V}, V, T), (\bar{V}, V, T)\}$.