Differential cross section of hard sphere collisions

Beads of negligible size are shot at a sample of thickness $x$ modeled as an amorphous collection (distributed with a volume density $n$) of hard spheres of radius $R$. When a collision takes place, the bead bounces off the sphere with an angle of emission equal to its angle of incidence $\beta$ on the sphere surface, such that it is deflected by a scattering angle $\theta = \pi - 2\beta$. Since the spheres are much more massive than the beads, we may neglect their recoil.

(a) Without doing any calculations, what is the cross section $\sigma_{Tot}$ for the bead to be deflected by any non-zero angle? Explain.

(b) What is the cross section $\sigma_{2\pi/3}$ for the shot to be deflected by more than $2\pi/3$?

(c) Establish the relation between the impact parameter $b$ and the scattering angle $\theta$ for a collision of a bead with one of the hard spheres.

(d) Establish the cross section $\frac{d\sigma}{d\Omega}(\theta)$ for the bead to be deflected by an angle $\theta$.

(e) Check that your answers to questions (a) and (b) can be reproduced by performing an integration of your answer to question (d).

(f) A beam of beads with a cross section area $A$ and a flux density $J$ is aimed at the target. Express the rate at which beads will be observed to be scattered between angles $\theta_1$ and $\theta_2$.

(g) Estimate the maximal number density $n_{Max}$ for your calculation to be in relative error smaller than $\epsilon$ (Hint: A scattered bead has a non zero chance to scatter again in the sample before emerging. I am looking for an estimate only so do not worry about the beads scattered at an angle $\theta$ close to $\pi/2$).
[2] Maximal energy transfer
Consider a particle of mass $M$ and momentum $p$ which scatters off a particle of mass $m$ initially at rest in the laboratory frame. Establish the maximum energy transfer $T_{Max}$ between the particles in terms of their masses and in terms of the velocity $\beta_0$ and Lorentz factor $\gamma_0$ of the incident particle in the laboratory frame.

Using Bethe’s formula, estimate the thickness a slab of steel need to have to stop muons $\mu^-$ of energy 13 GeV.

[4] Photo-multiplier tube
A photo multiplier with a quantum efficiency of 15% is operated with a gain of $3 \times 10^6$. The output of the photomultiplier drives a 50 $\Omega$ resistor. The voltage across the resistor is displayed by an oscilloscope. An individual photo electron is observed to produce a pulse of width 5 ns

(a) Estimate the single photo electron pulse amplitude displayed on the oscilloscope.

(b) When the photomultiplier is exposed to a constant source of light, a DC voltage $V = 12$ mV appears on the oscilloscope. Estimate the number of photons reaching the photocathode per unit time.

[5] Chance probability for a photon to make it through the atmosphere
Estimate the chance probability for an astrophysical 100 MeV gamma ray to reach the top of the mountain with a vertical incidence without interacting in the atmosphere. How would your result change for an incidence 60° from vertical? Consider the atmospheric pressure at the mountain top to be 60 kPa and the atmosphere composed of Nitrogen 80% and Oxygen 20%.

[6] Paraffin as a neutron shield
Paraffin $C_n H_{2n+2}$ have a mass density $\rho = 900$ kgm$^{-3}$. It is a hydrogen rich material which is effectively used as a shield against neutrons. Estimate the thickness of paraffin needed to attenuate a neutron beam by a factor 100.