Statistical properties of the collision time interval

Consider an electron chosen at a time \( t = 0 \). The probability for the electron to undergo scattering within any infinitesimal time \( dt \) is \( dp = \frac{dt}{\tau} \). You may find the answers to some of the questions to be self-evident but, still, go through the argumentation and give the detail of the calculation taking you to the answer.

(a) Derive the probability density \( \frac{dP}{dt} \) of the time \( t \) until the first collision after the electron is picked? (Hint: you divide divide the time interval \( t \) in small steps \( \delta t \ll \tau \) to conquer)

(b) What is the average time \( \langle t \rangle \) until the first collision after the electron is picked?

(c) What is the average time \( \langle t \rangle \) since the last collision before the electron is picked?

(d) An electron goes through two successive collisions at times \( t_1 \) and \( t_2 \) respectively. Consider \( T = t_2 - t_1 \) and derive the probability density for \( T \) (Note that "derive" is typed in bold font. It means you are expected to integrate probability densities over all the possible combinations of \( t_1 \) and \( t_2 \) with \( t_2 > t_1 \)).

(e) Starting from your answer to question (d), establish the average time interval between two successive collisions.

(f) Your answers to questions (b) and (c) suggest the time interval between two successive collisions is twice your answer to the previous question. Explain why this is not the case.

(g) What is the standard deviation \( \sigma_\tau \) of the time until the first collision after the electron is picked?

(h) Derive the probability \( P(n,T) \) for a given electron to undergo \( n \) collisions in a given time interval of width \( T \). (Hint: Here again, you should divide the time interval \( T \) in \( N \) small steps \( \delta t \ll \tau \) and express the probability as a binomial distribution. Then, you can take the limit \( N \to \infty \))