Physics 5520 : Solid State Physics II

Problem sheet 5. Out: April 2nd, due in: April 9th, 4 pm (post in Kipp's box if not in class). Instructions: Write your name/ID on the top of each sheet. Staple all sheets together, or else they will be lost! Let us follow your line of thought, i.e. provide some details, please. There are a total of 10+3 points.

1. Wannier excitons in GaAs (4 points)

Consider a Wannier-Mott exciton in the III-V semiconductor GaAs. The effective mass of the electron is $m_e=0.067m_0$, that of the heavy hole is $m_{hh}=0.34m_0$. Also, $\varepsilon_\infty=13$.

a) There are three valence bands in GaAs. Why do we consider the heavy hole band?
b) Compute the Bohr radius a_B and the binding energy E_B of the excitons.
c) The exciton is in a 1s state with a kinetic energy of 20 meV. Compute its speed (group velocity) in this state.
d) Can the exciton in this state (problem c) undergo direct, radiative recombination, without any preceding relaxation? Consider the dispersion relation of light. The band gap of GaAs at low temperatures is 1.518 eV.

2. Two level system in a magnetic field (3 points)

Consider a system of atoms with a total angular momentum quantum number of *J*. The atoms are exposed to a magnetic field of strength B_0 at temperature T. Under the premise that there is no interaction between the magnetic moments of the atoms you can calculate the average magnetic moment $\langle\mu_z\rangle$ of an atom in z-direction using the Boltzmann statistics for distinguishable particles:

$$\langle \mu_z \rangle = -g\mu_B \frac{\sum_{m_J=-J}^{+J} m_J \exp(-m_J\eta)}{\sum_{m_J=-J}^{+J} \exp(-m_J\eta)},$$ where we use the dimensionless parameter $\eta=g\mu_B B_0/(k_B T)$

which gives the ratio between magnetic and thermal interaction energy.

Consider for the simplest case a "two level system", with *J*=1/2. An example of this is the paramagnetic ion Cu^{2+} (what happens here is that the crystal field of a Cu^{2+} crystal, i.e. the average interaction of the atoms, leads to an effective annihilation of orbital angular momentum, so that L=0).

a) Compute the average magnetic moment $\langle\mu_z\rangle$ in a field B_0. Plot $\langle\mu_z\rangle/\mu_B$ against η and provide a brief discussion of the result.
b) Can you imagine what may be meant by "*cooling through adiabatic demagnetization*"? Provide a discussion in the context of this problem and briefly illustrate how you may construct a cryostat using Cu^{2+} as a cryogen.

3. Magnetic contribution to heat capacity (3 points)

A system of *N* non-interacting spins with S=½ (and L=0) is placed in a magnetic field of strength B_0. Show that there is a contribution to the specific heat capacity of the material of

$$C_M = Nk_B(2\mu_B B_0/k_B T)^2 \frac{\exp(2\mu_B B_0/k_B T)}{(1+\exp(2\mu_B B_0/k_B T))^2}.$$

This is often referred to as the *Schottky specific heat anomaly*. Why do you think Mr. Schottky was interested in this? (think back of our discussion of Schottky defects). Plot the function against reduced temperature $t=k_BT/2\mu_BB_0$. Does it remind you of anything? If you've taken statistical physics, it should.

4*. Why do frogs fly? – solids in an inhomogeneous magnetic field (3 points)

Compute the force acting on a solid situated in a spatially inhomogeneous magnetic field. Assume that a constant homogeneous magnetic field B_0 is superimposed by a field gradient dB/dx. Discuss the change in free energy (F=U-TS) by moving the object from position x to $x+dx$. What is the experimental relevance of this effect in determining the magnetic susceptibility (hint: you may want to look up the *Faraday balance*).