Physics 5520 : Solid State Physics II

Problem sheet 7. Out: April 18th, due in: April 25th, 4 pm (post in Kipp's box if not in class). Instructions: Write your name/ID on the top of each sheet. Staple all sheets together, or else they will be lost! Let us follow your line of thought, i.e. provide some details, please. There are a total of 10+3 points.

1. London equations (4 points)

a) Using the "zero resistance" property of superconductors, justify the second London equation relating electric field to current density, $E = \Lambda \frac{\partial J}{\partial t}$, and show that $\Lambda = m/ne^2$.

b) The penetration depth of Hg (T_c=4 K) at 3.5 K is about 75 nm. Estimate the values of the penetration depth and the number of superelectrons per unit volume at 1 K.

2. Explosive superconducting magnets (2 points)

A superconducting solenoid cooled in liquid helium has 10,000 turns and is 10 cm long. Its mean diameter is 3 cm. The current through the solenoid is increased slowly and when it reaches 20 A the solenoid quenches (i.e. it ceases to be superconducting – with catastrophic consequences as it heats up, suddenly leading to rapid evaporation of helium). Estimate the volume of liquid helium which is evaporated (latent heat of liquid helium: 2.5 J/cm^3).

3. Fluxons (4 points)

a) A Bitter pattern (i.e. the spatial arrangement of iron filings, think back of the movies we watched) is produced for a Type II superconductor which was found to consist of black spots approximately 2 microns apart. Estimate the field applied while making the Bitter pattern.

b) Using dimensionality arguments find a value for the force per unit length between two fluxons that are less than the penetration depth apart in a superconductor where the lower critical field is 10 mT. You may assume that the peak field in a fluxon is $2\mu_0 H_{C1}$ and that this field decays over a characteristic length λ - the penetration depth.

4.* Meissner effect in a sphere (3 points)

Consider a sphere of a Type I superconductor with critical field H_c.

a) Show that in the Meissner regime the effective magnetization M within the sphere is given by -8M/3)=B_a, where B_a is the uniform applied magnetic field.

b) Show that the magnetic field at the surface of the sphere in the equatorial plane is $3B_a/2$. It follows that the applied field at which the Meissner effect starts to break down is $2H_c/3$. (Hint: the demagnetization field of a uniformly magnetized sphere is $-4\pi M/3$)