When electron revolves around a circular orbit of radius $R$, the dipole moment of the system is $\vec{p} = -e \vec{r}$ where vector $\vec{r}$ connects a proton and an electron.

$\vec{p} = -e \vec{r}$ Acceleration $\vec{r}$ is given by the Coulomb law:

$$m \ddot{\vec{r}} = -\frac{e^2}{4\pi\varepsilon_0} \frac{\vec{r}}{R^3}$$

$m$-electron mass

so that $\vec{p} = \frac{e^3 R}{4\pi\varepsilon_0 R^3 m}$ and $1\ddot{\vec{r}} = \frac{e^3}{4\pi\varepsilon_0 R^2 m}$

According to Eq. (11.60) the rate of loss of energy of the system due to radiation is

$$\frac{dw}{dt} = -\frac{\mu_0 P^2}{6\pi c} = -\frac{\mu_0 e^6}{6\pi(4\pi\varepsilon_0)^2} \frac{1}{R^4 \text{cm}^2}$$

The energy stored in the system $W = W_p + W_k$ consists of potential energy

$$W_p = -\frac{e^2}{4\pi\varepsilon_0 R}$$

and kinetic energy

$$W_k = \frac{mv^2}{2}$$
which with the help of the Newton's law

\[ \frac{mv^2}{R} = \frac{e^2}{4\pi\varepsilon_0 R} \]

is \[ W_k = -\frac{W_0}{2} \]

so that \[ W = W_k + W_p = \frac{W_0}{2} = -\frac{e^2}{8\pi\varepsilon_0 R} \]

The relation between the rate of energy change and the rate at which \( R \) changes is found from differentiating this expression:

\[ \frac{dw}{dt} = \frac{e^2}{8\pi\varepsilon_0 R^2} \frac{dr}{dt} \]

Thus,

\[ \frac{e^2}{8\pi\varepsilon_0 R^2} \frac{dr}{dt} = -\frac{\mu_e e^6}{6\pi(4\pi\varepsilon_0)^2 R^4} \frac{1}{cm^2} \]

or \[ \frac{dr}{dt} = -\frac{\alpha}{R^2} \]

where \[ \alpha = \frac{\mu_e e^4}{12\pi^2 \varepsilon_0 cm^2} \]

Solution to this equation is found as follows:

\[ R^2 \frac{dr}{dt} = -\alpha dt \Rightarrow \frac{R^3}{3} = \text{const} - \alpha t \]

\[ \text{const} = \frac{R_0^3}{3} \quad (R_0 - \text{initial radius}) \]

\[ \frac{R^3}{3} = \frac{R_0^3}{3} - \alpha t \]

The 'collapse' happens when \( R(t) = 0 \)

which gives

\[ t = \frac{R_0^3}{3\alpha} = \frac{4\pi^2 \varepsilon_0 C R_0^5 m^2}{\mu_e e^4} \]
or, excluding $\mu_0$ by using $\mu_0 = \frac{1}{\varepsilon_0 c^2}$

$$t = \frac{4\pi^2 \varepsilon_0 c R_0 m^2}{e^4}$$

For $R_0 = 1 \times 10^{-10} m$, $m = 9.1 \times 10^{-31} kg$

$e = 1.6 \times 10^{-19} C$

we find

$$t = \frac{4 \cdot (3.14)^2 (8.85 \cdot 10^{-12})^2 (3.10^8)^3 (10^{-10})^3 (9.1 \cdot 10^{-31})^2}{(1.6 \cdot 10^{-19})^4}$$

$$= \frac{4 \cdot (3.14)^2 (8.85)^2 \cdot (9.1)^2 \cdot 10^{-24} \cdot 10^6 \cdot 10^{-60}}{(1.6)^4} \cdot 10^{-76}$$

$$= 1.05 \cdot 10^{-6} \cdot 10^{-15} s \approx 1.10^{-9} s = 1 \text{ nsec}$$