Selecting a cylindrical surface of radius $r$ and length $L$ we can write Gauss's theorem

$$E \cdot 2\pi r \cdot L = \frac{L}{\varepsilon_0} \left\{ \begin{array}{ll} \pi r^2 \rho, & r < R \\ \pi R^2 \rho, & r > R \end{array} \right.$$ 

We thus obtain

$$E(r) = \left\{ \begin{array}{ll} \frac{r \rho}{2 \varepsilon_0}, & r < R \\ \frac{R^2 \rho}{2 \varepsilon_0 r}, & r > R \end{array} \right.$$ 

2. The field inside the hollow can be represented as the difference of the field $\vec{E}_1$ created by the big sphere without hollow and of the field $\vec{E}_2$ of the hollow filled with the same charge density $\rho$. The field inside the sphere at a point $\vec{r}$ from its center is

$$\vec{E}_i = \frac{\rho}{3 \varepsilon_0} \vec{r}$$
Similarly, the field at the filled hollow would be

\[ \vec{E}_2 = \frac{P}{3\varepsilon_0} \left( \vec{r} - \vec{a} \right) \]

this is distance to the center of hollow

The difference is therefore

\[ \vec{E} = \vec{E}_1 - \vec{E}_2 = \frac{P\vec{a}}{3\varepsilon_0} \]

and does not depend on where exactly inside the hollow it is measured.

\[ \text{(3)} \]

From symmetry it is clear that the field is directed along the diagonal OA and that two 'legs' contribute the same amount to it.

It is, thus, sufficient to calculate projection of the field created by e.g. lower leg onto the diagonal direction.

Charge between \( \theta \) and \( \theta + d\theta \) is

\[ dq = \lambda \frac{OB \, d\theta}{\cos \theta}, \quad OB = \frac{L}{\cos \theta} \]

Field of this charge at point O

\[ dE = \frac{1}{4\pi \varepsilon_0} \frac{\lambda \, d\theta}{(OB)^2} = \frac{\lambda \, d\theta}{4\pi \varepsilon_0 \, OB \, \cos \theta} = \frac{\lambda \, d\theta}{4\pi \varepsilon_0 \, L} \]
And its projection onto \( A_0 \)

\[ dE_{A_0} = dE \cos\left(\frac{\pi}{4} - \theta\right) \]

the total field is given by the integral from 0 to \( \frac{\pi}{4} \) times factor 2:

\[
E = 2 \frac{\lambda}{4 \pi \varepsilon_0 l} \int_0^{\frac{\pi}{4}} d\theta \cos\left(\frac{\pi}{4} - \theta\right)
\]

\[
\left. \sin\left(\frac{\pi}{4} - \theta\right) \right|_0^{\frac{\pi}{4}} = \frac{1}{\sqrt{2}}
\]

and

\[
E = \frac{\lambda}{2 \pi \varepsilon_0 l}
\]

Field is pointed along the axis.

The ring between \( r \) and \( r + dr \) contributes to the axial component

\[ dE = \frac{Q}{\pi \varepsilon_0} \frac{1}{4 \pi \varepsilon_0} \frac{2 \pi \varepsilon_0}{2} \cos \theta \]

surface charge density

\[ \cos \theta = \frac{x}{\sqrt{x^2 + r^2}} \]

Taking the integral

\[
E = \frac{Q x}{2 \pi R^2 \varepsilon_0} \int_0^R \frac{r dr}{(x^2 + r^2)^{3/2}} = \frac{Q}{2 \pi \varepsilon_0} \frac{1}{\sqrt{x^2 + r^2}} \left|_0^R \right.
\]

\[
= \frac{Q}{2 \pi R^2 \varepsilon_0} \left( 1 - \frac{x}{\sqrt{x^2 + R^2}} \right)
\]
a) for \( x \gg r \), 
\[ \frac{x}{\sqrt{x^2 + k^2}} \approx 1 - \frac{r}{2x^2} \]

and 
\[ E \approx \frac{Q}{4\pi \varepsilon_0 x^2} \quad \text{(field of point charge)} \]

b) for \( x \ll r \), 
\[ E \approx \frac{Q}{2\pi \varepsilon_0 R^2} \quad \text{(field of infinite plane with surface charge \( Q/R^2 \))} \]