Homework 9: solutions

1. \( a \)
   \[ B(r) = \frac{\mu_0 I}{2\pi r} \]
   \[ \phi = a \int_0^\infty B(r) dr = a \frac{\mu_0 I}{2\pi} \ln \frac{S+a}{S} \]
   \[ \text{Direction: counterclockwise.} \]
   \[ \text{b) } \epsilon = -\frac{d\phi}{dt} = -\frac{a\mu_0 I}{2\pi} \left( \frac{1}{S+a} - \frac{1}{S} \right) \frac{ds}{dt} = \]
   \[ \frac{d^2 I}{dt^2} \]
   \[ \text{c) No change of flux \rightarrow no EMF.} \]

2. \( \text{a) } B = \mu_0 I n, \ \text{flux} \ \Phi = \mu_0 I n \pi a^2 \]
   \[ |\epsilon| = \mu_0 n a^2 \frac{dI}{dt} = \frac{\mu_0 n a^2}{R} \Rightarrow I_R = \frac{\mu_0 n a^2}{R} \]
   \[ \text{Direction: Left (opposite to the currents in the loops)} \]
   \[ \text{b) Change of flux:} \]
   \[ \Delta \Phi = 2\mu_0 n a^2 I \]
   \[ \text{Change: } \Delta Q = \epsilon \Delta t \frac{1}{R} = \frac{\Delta \Phi}{R} = \frac{2\mu_0 n a^2 I}{R} \]
   \[ \text{Direction: right (clockwise)} \]

3. \[ B = 0 \]
   \[ \int ds \]
Electric field is found from
\[ E \cdot 2\pi r = \frac{dB}{dt} \pi r^2 \Rightarrow E = \frac{r}{2} \frac{dB}{dt} \]

Increase in velocity:
\[ dv = \frac{qE dt}{m} = \frac{qr}{2m} dB \]

Increase in kinetic energy, \[ dt = mvdv = \frac{qr^2}{2} B \]

The total centripetal acceleration is now provided by both the Coulomb force and Lorentz force:
\[ a = \frac{qE}{\varepsilon_0 r^2 m} + \frac{qvdB}{m} \Rightarrow da = \frac{qvdB}{m} \]

On the other hand, \[ a = \frac{v^2}{r} \], and
\[ da = \frac{2vdv}{r} = \frac{2v}{r} \frac{qr}{2m} dB = \frac{qvdB}{m} \]

and the two expressions coincide, indicating that \( r \) is indeed constant.