Problem 23. The number of molecules traveling within the angle interval $d\theta$ per unit volume is $n \sin \theta d\theta$ the number of such molecules reaching the hole during time $\Delta t$ is

$$\frac{1}{2} n \sin \theta d\theta \times \bar{v} \cos \theta A \Delta t.$$ 

Integrating over all angles from 0 to $\pi/2$ we find the total number of molecules lost,

$$N_{\text{lost}} = \frac{1}{4} n \bar{v} A \Delta t$$

Concentration $n$ can be expressed via temperature and pressure with the help of the equation of state, $n = P/k_B T$, and average velocity from the equipartition theorem, $\bar{v} = \sqrt{\frac{3k_B T}{m}}$ (more precisely, from the Maxwell distribution, $\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}$). Finally, relating the number of molecules to the number of moles $M_{\text{lost}} = N_{\text{lost}}/N_A$, and the molecular mass $m$ to the molar mass $\mu = m N_A$, we obtain,

$$M_{\text{lost}} = \frac{\sqrt{3}PA\Delta t}{4\sqrt{\mu RT}}.$$ 

For estimates we use $\mu = 0.03 kg/mole$, $\Delta t = 60 s$, $P = 10^5 Pa$, $T = 300 K$, $A = 10^{-7} cm^2 = 10^{-11} m^2$,

$$M_{\text{lost}} = 3 \times 10^{-6} moles.$$ 

This approximately corresponds to $30 cm^3$ of air.

Problem 24. If the two fluxes (from left to right and from right to left) are equal to each other, then $n_1 \bar{v}_1 = n_2 \bar{v}_2$, where indices $L$ and $R$ indicate quantities related to the left and right reservoirs. If gases consist of the same molecules, then

$$n_1 \sqrt{T_1} = n_2 \sqrt{T_2}.$$ 

Using the equation of state, $P = nk_B T$ we can relate the two pressures:

$$P_1 = P_2 \sqrt{\frac{T_1}{T_2}}.$$ 

This is obviously different (!) from the thermodynamic condition of equilibrium $P_1 = P_2$. Which one is correct?

Problem 25. To find surface tension

$$\sigma = \frac{\Delta p}{L \Delta t},$$

we need to find the total momentum $\delta p$ transferred to the length of boundary $L$ during time $\Delta t$. The fraction of molecules traveling within the angle interval $d\phi$ is given in two dimensions simply
by $d\phi/2\pi$ (note the difference with the usual 3D case – there is only one angle in 2D). Total number of molecule traveling within the interval $d\phi$ per unit of area is then equal

\[ \frac{N d\phi}{2\pi}. \]

During time $\delta t$ all the molecules that are within the area $v\Delta t L \cos \phi$ will hit the boundary. Each collision will transfer momentum $2mv \cos \phi$ to the boundary. The total momentum transfer is, therefore, given by,

\[ \delta P = \pi/2 \int_{-\pi/2}^{\pi/2} \frac{N d\phi}{2\pi} \times v\Delta t L \cos \phi \times 2mv \cos \phi. \]

Calculating the angle integral and averaging over the molecular velocities (which is done simply by changing $v^2 \rightarrow \bar{v}^2$) we obtain,

\[ \Delta p = \frac{1}{2} \bar{v}^2 NL \Delta t, \quad \rightarrow \quad \sigma = \frac{1}{2} \bar{v}^2 N. \]

Using equipartition theorem (and remembering that there are only two degrees of freedom in 2D), $m\bar{v}^2/2 = k_B T$, the surface tension acquires the form, $\sigma = k_B NT$ which resembles the usual equation of state of ideal gas.