Towards optical intensity interferometry for high angular resolution stellar astrophysics

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Motivations.
Background.
Cherenkov telescope arrays.
The imaging problem: Phase recovery.
Simulations of future intensity interferometers.
Experimental efforts.
Conclusions.
Motivations for stellar intensity interferometry

0.1 mas resolution
\( \lambda = 400 \text{nm} \)

(Simulation by B. Freitag)

\( \sim 1 \text{ km baseline} \)

- Disk-like stars: Diameters at different wavelengths.
- Rotating stars: Oblateness and pole brightening.
- Interacting binaries.
- Mass loss and mass transfer.

Altair (CHARA)

\( \alpha \) – Arae (VLTI)
Complex Degree of Coherence $\gamma$

$$\gamma(r_i, t_i; r_j, t_j) \equiv \frac{\langle E^*(r_i, t_i) \cdot E(r_j, t_j) \rangle}{\sqrt{\langle |E^*(r_i, t_i)|^2 \rangle \langle |E^*(r_j, t_j)|^2 \rangle}}$$

**Amplitude Interferometry**

$$\gamma(x) = \mathcal{F}[O(\theta)]$$

**Intensity Interferometry (I.I.)**

$$|\gamma(x)|^2 = |\mathcal{F}[O(\theta)]|^2$$
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Intensity Interferometry (I.I.)

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Correlator

$$C_{ij} = \frac{\langle \Delta I_i \Delta I_j \rangle}{\langle I_i \rangle \langle I_j \rangle}$$

Analysis
Intensity Interferometry

\[ C_{ij} = \frac{\langle \Delta I_i \Delta I_j \rangle}{\langle I_i \rangle \langle I_j \rangle} \]

- Hanbury Brown & Twiss. 1950’s
- Correlation of Intensities.
- No need for sub-wavelength precision.
- Insensitive to turbulence.

Narrabri 1963
Intensity Interferometry

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Narrabri 1963
Gamma-ray astronomy

Primary $\gamma$ ray initiates electromagnetic cascade, which in turn emits Cherenkov radiation.

VERITAS array in southern arizona
Be star and compact companion.

TeV emission is maximum at apastron.

TeV attenuation due to interactions with stellar photons and circumstellar hydrogen.

Compact object is a black hole.

X-ray binary LSI + 61°303

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The Cherenkov Telescope Array (CTA)

- ACTs are ideal SII receivers.
- Easily adapted for optical SII ($\lambda \sim 400 \text{ nm}$)
- Future arrays will provide thousands of simultaneous baselines.
- SII working group in CTA.
Array design and \((u, \nu)\) coverage

\[
\lambda = 400 \text{ nm}
\]

\[
\Delta \theta_{\text{min}} \sim \frac{\lambda}{\Delta x_{\text{max}}} \sim 0.01 \text{ mas}
\]

\[
\Delta \theta_{\text{max}} \sim \frac{\lambda}{\Delta x_{\text{min}}} \sim 1 \text{ mas}
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Data simulation

0.1 mas, $m_v = 3$, 10 hrs (Nuñez et al. 2012)
The imaging problem

\[ \gamma \sim \text{Fourier transform of the radiance distribution of the star.} \]

\[ \gamma(x) \sim \int O(\theta) e^{ikx\theta} d\theta \]

- In SII we have access to the squared modulus of the complex degree of coherence $|\gamma|^2$
- When $|\gamma|^2$ is measured, the phase of the Fourier transform is lost.
- How can we recover images?
Some alternatives?

- Model fitting.
- Maximum likelihood approach. (Thiebaut 2006).

Phase recovery

- Three point correlations. $\langle I_i I_j I_k \rangle$ (Ribak 2006)
- Analytic approach. (Holmes 2004).
  The Fourier transform of an object of finite size (e.g. a star) is an analytic function.
  Relate magnitude to phase through Cauchy-Riemann equations.
Some alternatives?

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  The Fourier transform of an object of finite size (e.g. a star) is an analytic function.
  Relate magnitude to phase through Cauchy-Riemann equations.
Cauchy-Riemann phase recovery in 1-D  
(Holmes 2004)

\[
\gamma(z) = \sum_j l(j\Delta\theta)z^j \quad ; \quad z \equiv \exp(i \, mk \Delta x \Delta \theta) \equiv e^{i\phi}
\]

\[
z \equiv \xi + i\psi \quad ; \quad \gamma \equiv \text{Re}^{i\phi}
\]

\[
\frac{\partial \Phi}{\partial \psi} = \frac{\partial \ln R}{\partial \xi} \equiv \frac{\partial s}{\partial \xi}
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\frac{\partial \Phi}{\partial \xi} = -\frac{\partial \ln R}{\partial \psi} \equiv -\frac{\partial s}{\partial \psi}
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\[
\Delta s_{||} = \frac{\partial s}{\partial \xi} \Delta \xi + \frac{\partial s}{\partial \psi} \Delta \psi = \frac{\partial \Phi}{\partial \psi} \Delta \xi - \frac{\partial \Phi}{\partial \xi} \Delta \psi \equiv \Delta \Phi_{\perp}
\]

- Log-Magnitude differences $\leftrightarrow$ Phase differences.
Cauchy-Riemann phase recovery in 1-D (Holmes 2004)

\[ \gamma(z) = \sum_j l(j\Delta\theta)z^j ; \quad z \equiv \exp(i \, mk \Delta x \Delta \theta) \equiv e^{i\phi} \]

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- Log-Magnitude differences $\leftrightarrow$ Phase differences.
1D example

**Magnitude**

**Phase**

**Image**
1D examples
Simulation/Analysis overview

- **Pristine**
- **Data**
- **Hermite fit**
- **MiRA**
- **Gerchberg-Saxton**
- **Cauchy-Riemann**

**arg min \{ \chi^2 \}**

Thiebaut 2006
Simulation/Analysis overview

Pristine \rightarrow Data \rightarrow Hermite fit

MiRA \leftarrow Gerchberg-Saxton \leftarrow Cauchy-Riemann

\arg \min \left\{ \chi^2 \right\}

Thiebaut 2006

\frac{\partial \Phi}{\partial \psi} = -\frac{\partial \ln R}{\partial \psi}

\frac{\partial \Phi}{\partial \xi} = \frac{\partial \ln R}{\partial \xi}

\partial \Phi \partial \Omega_k = M_k e^{i \phi_k}

\partial \Phi \partial M_k = |\gamma|
Introduction

ACTs and γ-ray astronomy

SII with IACTs

The imaging problem

Experimental efforts

Closing

Nunez, et. al. 2012,

MNRAS, 419, 172
Post-processing example: Star spots

\[ m_\nu = 3, \text{ 10 hrs of observation} \]

\[ \Delta T = 500^\circ\text{K} \]

Post-processing example: Star spots

Effect of finite telescope size?

electronic noise, stray light?

Rou et al. in prep.
Attempts to measure $|\gamma|^2$ in the lab

Light source → Pinhole → BS → PMT$_1$ → Correlator

Experimental efforts

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Introduction
Attempts to measure $|\gamma|^2$ in the lab

- Light source
- Pinhole
- Artificial star
- BS
- PMT$_1$
- PMT$_2$
- Correlator
Attempts to measure $|\gamma|^2$ in the lab

- Light source
- Pinhole
- Artificial star
- BS
- PMT$_1$
- PMT$_2$
- Correlator

Miniature telescopes
Attempts to measure $|\gamma|^2$ in the lab

- Light source
- Pinhole
- Artificial star
- Miniature telescopes
- Analog or digital
- Correlator
- PMT$_1$
- PMT$_2$
- BS

David Kieda
Attempts to measure $|\gamma|^2$ in the lab

- Light source
- Pinhole
- Artificial star
- BS
- PMT$_1$
- PMT$_2$
- Correlator

Diagram showing the experimental setup with light source, pinhole, and correlator.
Attempts to measure $|\gamma|^2$ in the lab

\[
\theta \approx \frac{1 \times 10^{-4} \text{m}}{3 \text{m}} = 10^{-4} \text{rad}
\]

\[
\theta = 1.22 \frac{\lambda}{d}
\]

\[
\Rightarrow d = 6.5 \text{ mm}
\]
Results with the pseudo-thermal light source

![Graph showing baseline versus $|\gamma|^2$ for different baselines (0.2 mm, 0.3 mm, 0.5 mm).](image-url)
Results with the pseudo-thermal light source

\[ d = 1.22 \lambda / \theta \]
Imaging from autocorrelation data

In collaboration with Ryan Price and Erik Johnson.
Imaging from autocorrelation data

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Imaging from autocorrelation data

Laser → Beam expander → Mask → Rough surface

CPU → CCD

Speckle image
Imaging from autocorrelation data

In collaboration with Ryan Price and Erik Johnson.
Imaging from autocorrelation data

Laser → Beam expander → Mask → Rough surface

Speckle image → Autocorrelation → Reconstruction

In collaboration with Ryan Price and Erik Johnson.
Imaging from autocorrelation data
Starting observations of Spica

0.53 mas
0.22 mas
1.8 mas

4.0 day period
\( \lambda = 400 \text{nm} \)

StarBase at transit

Squared degree of coherence

Inter-telescope distance (m)
Starting observations of Spica

- Analyzed 1s of data.
- Need several hours of data to detect a correlation.
- Expect to see modulation of correlation as a function of time.
Conclusions

- Fast electronics and IACT arrays will allow to measure $|\gamma|^2$
- SII will provide high angular resolution images of stars.
- Advantages when compared to amplitude interferometry.
- Phase can be recovered via several techniques.
- Images can be used for stellar astrophysics.
- Experimental efforts allow to understand the advantages and difficulties of modern SII.
Thanks to:

- Stephan LeBohec, Dave Kieda and Janvida Rou.
- Lina Peralta.
- Physics dept: Jackie Hadley, Heidi Frank.
- Michelle Hui, Gary Finnegan, Nick Borys ...
- ...

THE END
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THE END
JUST IN CASE
Two component Fourier model (Hanbury Brown 1974)
Detected currents at A and B

\[ i_A = K_A (E_1 \sin(\omega_1 t + \phi_1) + E_2 \sin(\omega_2 t + \phi_2))^2 \]

\[ i_B = K_B (E_1 \sin(\omega_1 (t + d_1/c) + \phi_1) + E_2 \sin(\omega_2 (t + d_2/c) + \phi_2))^2 \]
Two component Fourier model (Hanbury Brown 1974)

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\[ i_B = K_B (E_1 \sin(\omega_1 (t + d_1/c) + \phi_1) + E_2 \sin(\omega_2 (t + d_2/c) + \phi_2))^2 \]

Band pass of electronics \( \Delta f \approx 100\,\text{MHz} \)

\[ i_A = K_A E_1 E_2 \cos((\omega_1 - \omega_2) t + (\phi_1 - \phi_2)) \]

\[ i_B = K_B E_1 E_2 \cos((\omega_1 - \omega_2) t + (\phi_1 - \phi_2) + \omega_1 d_1/c - \omega_2 d_2/c) \]

\( i_A \) and \( i_B \) are correlated.
Intensity fluctuations and correlations

$$\langle \Delta n^2 \rangle = \langle n \rangle + \langle n \rangle^2 \frac{\tau_c}{T}$$

- **Shot noise**
- **Wave noise**

Wave noise is small but correlated between neighboring detectors.
Cauchy-Riemann phase recovery (1D)

\[
\gamma(m\Delta x) = \sum_j I(j\Delta \theta) \exp(i m k \Delta x j \Delta \theta)
\]

\[
\gamma[z(m\Delta x)] = \sum_j I(j\Delta \theta) z^j ; \quad z \equiv \exp(i m k \Delta x \Delta \theta) \equiv e^{i\phi}
\]

\[
z \equiv \xi + i\psi ; \quad \gamma \equiv \text{Re}^{i\Phi}
\] (1)
Cauchy-Riemann phase recovery

\[ \gamma(m\Delta x) = \sum_j l(j\Delta \theta) \exp(i \cdot mk\Delta x \cdot j\Delta \theta) \]  \hspace{1cm} (2)

\[ \gamma[z(m\Delta x)] = \sum_j l(j\Delta \theta)z^j ; \ z \equiv \exp(i \cdot mk\Delta x \Delta \theta) \equiv e^{i\phi} \]  \hspace{1cm} (3)

\[ z \equiv \xi + i\psi ; \gamma \equiv \text{Re}^{i\Phi} \]  \hspace{1cm} (4)

\[ \frac{\partial \Phi}{\partial \psi} = \frac{\partial \ln R}{\partial \xi} \equiv \frac{\partial s}{\partial \xi} \]  \hspace{1cm} (5)

\[ \frac{\partial \Phi}{\partial \xi} = -\frac{\partial \ln R}{\partial \psi} \equiv -\frac{\partial s}{\partial \psi} \]  \hspace{1cm} (6)
\[ \Delta s_\xi = \frac{\partial s}{\partial \xi} \Delta \xi = \frac{\partial \Phi}{\partial \psi} \Delta \xi \]  \hspace{1cm} (7)

\[ \Delta s_\psi = \frac{\partial s}{\partial \psi} \Delta \psi = -\frac{\partial \Phi}{\partial \xi} \Delta \psi \]  \hspace{1cm} (8)

\[ \Delta s_{||} = \frac{\partial s}{\partial \xi} \Delta \xi + \frac{\partial s}{\partial \psi} \Delta \psi = \frac{\partial \Phi}{\partial \xi} (-\Delta \psi) + \frac{\partial \Phi}{\partial \psi} \Delta \xi \equiv \Delta \Phi \perp \]  

Since \(|z| = 1 \Rightarrow we can only access to the log-magnitude \(s\) along the unit circle in the \(\xi - \psi\) plane.
Φ is in general a solution to the Laplace equation in the $\xi - \psi$ plane.

$$\Phi(\phi) = \sum_j a_j \cos(j\phi) + b_j \sin(j\phi)$$ \hspace{1cm} (10)

$$\Delta \Phi_\perp(\phi) = \sum_j \{ (1 - 2\sin(\Delta \phi/2))^j - 1 \} (a_j \cos(j\phi) + b_j \sin(j\phi))$$ \hspace{1cm} (11)

- There is a relation between the log-magnitude differences and the phase.
- There will be ambiguities in the phase were the magnitude is zero.
- Reconstructed phase will have an unknown gross piston and tilt.