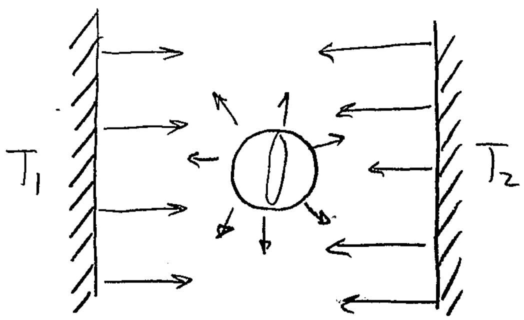
3760   2008 Final Solutions

Problem #1



P_{in} – power absorbed by the ball

$$P_{in} = \pi R^2 \cdot \sigma \left(T_1^4 + T_2^4 \right)$$

P_{em} – power emitted by the ball

$$P_{em} = \sigma (4\pi R^4) \cdot T_b^4$$

In equilibrium $\quad P_{in} = P_{em} \quad \Rightarrow$

$$T_b = \frac{1}{\sqrt{2}} \cdot \left(T_1^4 + T_2^4 \right)^{1/4}$$

Problem #2

Maxwell distribution in two dimension.

$$f(v_x, v_y)\, dv_x\, dv_y = A_2 \exp\left(-\frac{m v_x^2}{2kT}\right) \cdot \exp\left(-\frac{m v_y^2}{2kT}\right) \cdot dv_x\, dv_y.$$

A_2 – can be found from normalization $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(v_x, v_y)\, dv_x\, dv_y = 1$

or making analogy with 3-d case

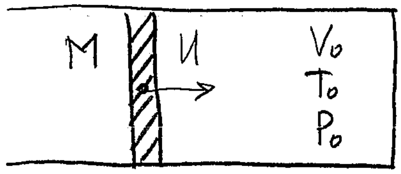
$$A_2 = \left(\frac{m}{2\pi kT}\right)$$

$$\langle v_x^+ \rangle = A_2 \int_0^{+\infty} v_x \exp\left(-\frac{m v_x^2}{2kT}\right) dv_x \int_{-\infty}^{+\infty} \exp\left(-\frac{m v_y^2}{kT}\right) dv_y =$$

$$= \left(\frac{m}{2\pi kT}\right)^{1/2} \cdot \int_0^{\infty} \left(\frac{2kT}{m}\right) \cdot \frac{1}{2} \cdot \exp\left(-\frac{m v_x^2}{2kT}\right) \cdot d\left(\frac{m v_x^2}{2kT}\right) =$$

$$= \left(\frac{kT}{m 2\pi}\right)^{1/2} \int_0^{\infty} \exp(-x)\, dx = \left(\frac{kT}{2\pi m}\right)^{1/2}$$

Problem #3



Kinetic energy of the piston is converted into work done on gas.

(1) $E_k = M\frac{U^2}{2}.$

(2) $W_{on\,gas} = \int_{V_0}^{V_f}(-P)dV$

System is thermally isolated => compression is adiabatic

$PV^\gamma = const$, gas is monoatomic => $\gamma = \frac{f+2}{f} = \frac{5}{3}$

(3) $W_{on} = -\int_{V_0}^{V_f} \frac{P_0 V_0^{5/3}}{V^{5/3}} dV = -P_0 V_0^{5/3} \cdot \left(-\frac{3}{2}\right)\frac{1}{V^{2/3}}\Big|_{V_0}^{V_f} =$

$= \left(\frac{P_0 V_0^{5/3}}{V_f^{2/3}} - P_0 V_0\right)\cdot\frac{3}{2}$

(4) For idabatic process $TV^{\gamma-1} = const$, for monoatomic gas.

$T\cdot V^{2/3} = const. = T_0 V_0^{2/3}$ => $\frac{1}{V_f^{2/3}} = \frac{T_f}{T_0 V_0^{2/3}}$

Combining (1) (2) and (4)

$M\frac{U^2}{2} = \frac{3}{2}\cdot\left(\frac{P_0 V_0}{T_0}\cdot T_f - P_0 V_0\right)$

$T_F = T_0\left(1 + \frac{MU^2}{3 P_0 V_0}\right)$

Problem # 4

A- homogeneous liquid. Composition $Au_{70}Ge_{30}$

B- heterogeneous mixture of liquid and solid phases.

solid – almost pure Ge

Liquid – $Au_{62.}Ge_{38}$

C- heterogeneous mixture of two solid phases.

1 solid – almost pure Ge

2nd solid – $Au_{98}Ge_{2}$.