Homework # 6

Problem #1 (5 points)
(Problem 3.34 from Schroeder with (d) simplified and some comments)
Polymers like rubber are made of very long molecules usually tangled up in configurations that have lots of entropy. As a very crude model of rubber band consider a chain of *N* links, each of the length ℓ (see figure 3.17 in book). Imagine that each links has only two possible states, pointing either left or right. The total length *L* of the rubber band is the net displacement from the beginning of the first link to the end of the last link.

(a) Find the expression for the entropy for the system in terms of N and N_R, the number of links pointing to the right. Write down the same formula in Gaussian approximation assuming N>>1
(b) Write down formula for L in terms of N and N_R.
(c) For one-dimensional systems as this the length *L* is analogous to the volume *V* of a three-dimensional system. Similarly, the pressure *P* is replaced by the tension force *F*. Taking *F* to be positive when the rubber band is pulling inward, write down and explain appropriate thermodynamic identity for this system. (Thermodynamic identity for ideal gas appears in p. 111 of Schroeder)

(d-e) Using the thermodynamic identity, you can now express the tension force *F* in terms of a partial derivative of the entropy. From this expression, using Gaussian approximation compute the tension in terms of L, T, N, ℓ

(f) Discuss the temperature dependence of the tension force on temperature. If you increase temperature of a rubber band, does it tend to expand or contract? Does this behavior make sense?

Problem #2 (5 points)
Problem #4-21 from Schroeder
If needed assume that you know a number of active degrees of freedom in the gas molecule.

Extra credit problem #1 (3 points)
The ideal heat engine woks in reverse cycle (so it is actually an ideal refrigerator). The machine transfer heat from a reservoir #1 with water at 0^oC to a reservoir #2with water at 100^oC. How much water does one need to freeze in the reservoir #1 to vaporize 1 kg of water in the reservoir #2. Look up in the textbook the latent heats of water vaporization and melting.

Extra credit problem #2 (4 points)
There are two bodies with initial temperatures T_1 and T_2 ($T_1 > T_2$) and heat capacities C_1 and C_2 that do not depend on temperature. (Heat capacity is for the whole body so if some heat is added to the body #1 then $\Delta Q_1 = C_1 \Delta T_1$). The body #1 is used as a heater and the body #2 as a cooler in an ideal heat engine. Find the maximum work you may get with such an engine. Compute this work when the body #1 is 1 kg of water at 100^oC and the body #2 is 1 kg of water at 0^oC.