Homework #8

Problem #1
A circular cylinder of radius R rotates about the long axis with angular velocity ω. The cylinder contains an ideal gas of atoms of mass *m* at temperature *T*. (The centrifugal force can be treated similar to the gravitational force in the atmosphere problem solved in class)

(a) Find an expression for the dependence of the concentration *n(r)* on the radial distance *r* from the axis, in terms of *n*(0) on the axis.
(b) What portion of the atoms in the cylinder is located in its central part between *r*=0 and *r=R/2*

Problem #2
A gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature *T*. Denote the velocity of a molecule as $\vec{v}$, its Cartesian components as v_x, v_y, v_z and its speed *v*. What are the following mean values:

(a) $\langle v_x \rangle$

(b) $\langle v_x^2 \rangle$

(c) $\langle v^2 v_x \rangle$

(d) $\langle v_x^3 v_y \rangle$

(e) $\left\langle \left(v_x + b v_y \right)^2 \right\rangle$ where *b* is constant

(f) $\langle v_x^2 v_y^2 \rangle$

Some comments and help:
$\langle\ \rangle$ -means averaging

Average of sum is equal to the sum of the averages: $\langle a+b \rangle = \langle a \rangle + \langle b \rangle$

(If you need to calculate explicitly any integral in this problem, you are the kind of person that like to turn cranks but does not think. All needed integrals are already computed and written in the lecture 11)

Problem #3
Monoatomic molecules (total number is *N*) adsorbed on a surface are free to move on this surface and can be treated as a classical two-dimensional gas. Temperature is *T*.

(a) Write an expression for a probability to find a molecule with the velocity in the range between $\vec{v}$ and $\vec{v} + d\vec{v}$ (v_x and $v_x + dv_x; v_y$ and $v_y + dv_y$) Evaluate normalization constant.
(b) Write an expression for a probability to find a molecule with the speed in the range between v and $v + dv$.
(c) What is the average number of molecules that have the speed in the range between v and $v + dv$?

(d) 3 point of extra-credit. What is the heat capacity of this “surface” gas of *N* molecules?