Homework #7

Pe #1 (6.15)

(a) $\langle E\rangle = \frac{1}{10}(4\times 0 + 3\times 1 + 2\times 4 + 1\times 6) = 1.7$ ev

(b) $P(0ev) = \frac{4}{10}$ $\quad P(1ev) = \frac{3}{10}$ $\quad P(4ev) = \frac{4}{10}$ $\quad P(6ev) = \frac{1}{10}$

(c) $\langle E\rangle = \sum E_s \cdot P_s = 0ev\cdot\frac{4}{10} + (1ev)\cdot\frac{3}{10} + 4(ev)\cdot\frac{2}{10} +$

$+ 6ev \cdot \frac{1}{10} = 1.7.ev.$

Pe #2 (6.17)

		ΔE_i^2
(a) 0ev	$\Delta E_1 = E_i - \langle E\rangle = -1.7ev$	2.89
1ev	$\Delta E_2 = -0.7ev$	0.49
4ev	$\Delta E_3 = +2.3ev$	5.29
6ev	$\Delta E_4 = +4.3ev$	18.49.

(b) $\langle \Delta E_i^2\rangle = \left(\sum_{i=1}^{10} \Delta E_i^2\right)\cdot\frac{1}{10} =$

$= \frac{1}{10}(11.56 + 1.47 + 10.58 + 18.49) = 4.21$

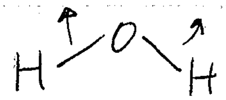
$\sigma_E = \sqrt{\langle E_i^2\rangle} = 2.05$

(c) $\sigma_E^2 = \frac{1}{N}\cdot\sum_i (E_i - \langle E\rangle)^2 = \underbrace{\frac{1}{N}\cdot\sum(E_i^2)}_{=\langle E^2\rangle} + \frac{1}{N}\sum_i(-2E_i\langle E\rangle + \langle E\rangle^2) =$

$= \langle E^2\rangle - \frac{1}{N}\langle E\rangle\cdot\left[\underbrace{\sum_i(E_i - \langle E\rangle)}_{=0} + \sum_i E_i\right] =$

$= \langle E^2\rangle - \langle E\rangle\cdot\frac{1}{N}\sum_i E_i = \langle E^2\rangle - \langle E\rangle^2$

Problem #3 (6.10)



(a) To estimate Z we compute Boltzmann factors for several first energy levels until contribution of high energy levels becomes negligible.

$T=300K$.

	E	B. factors	$\exp\left(-\frac{hf}{kT}\right) = 5\cdot10^{-4}$. $T=300k$.
0	$\frac{1}{2}hf$	$\left[\exp\left(-\frac{hf}{kT}\right)\right]^{1/2}$ =	0.022
1	$\frac{3}{2}hf$	$[\;—\text{"}—\;]^{3/2}$ =	$1\cdot10^{-5}$
2	$\frac{5}{2}hf$	$[\;—\text{"}—\;]^{5/2}$ ≈	$5.5\cdot10^{-9}$
3	$\frac{7}{2}hf$	$[\;—\text{"}—\;]^{7/2}$ =	

$Z = \sum B\text{ factors} \cong 0.022$. $\quad P(0) = \frac{(B.f)_1}{Z} = 1$.

$P(1) = \frac{1\cdot10^{-5}}{2.2\cdot10^{-2}} \cong 5\cdot10^{-4}$.

$P(2) \cong 2.5\cdot10^{-8}$

(b) $T=700k$. $\quad \exp\left(-\frac{hf}{kT}\right) = 3.8\cdot10^{-2}$.

$B(0) = 0.196$

$B(1) = 7.4\cdot10^{-3}$

$B(2) = 2.8\cdot10^{-4}$

$Z \approx 0.2037$

$P(1) = 3.6\cdot10^{-2}$.

$P(2) = 0.001$

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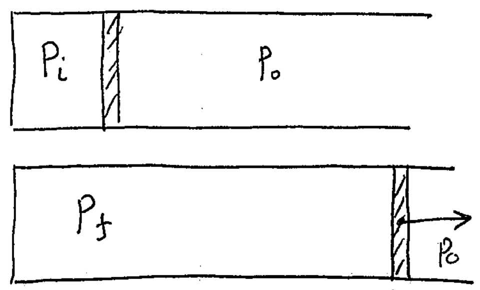
Pr # 3 extra-credit.



m – mass of the piston.

- Let's for simplicity assume that P_0 (pressure on the other side of the piston is much less than P_i and P_f $P_0 \ll P_i, P_f$.
- Ideal process – gas expands adiabatically all work done by the gas is converted into. kinetic energy of the piston

$$PV^{\gamma} = const \qquad PV = RT \qquad V_f / V_i = n \qquad \gamma = \frac{f+2}{f} = \frac{5}{3} \text{ (mono at. gas)}$$

$$TV^{2/3} = const. = T_i V_i^{2/3} = T_f V_f^{2/3} \qquad f = 3$$

$$(1) \qquad \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{2/3} = n^{2/3}$$ ← this is the best cooling we can obtain.

- The process is not ideal some friction in the system does exist. In extreme case all work done in expansion is converted into heat and kinetic energy of the piston does not change and equal to zero. In this case from 1st law.

$$\Delta U = \Delta Q - \Delta W = 0. \qquad \Delta W = \Delta Q$$ because system is thermally isolated

$$\Delta U = 0 \Rightarrow \Delta T = 0 \text{ (for ideal gas)} \Rightarrow$$

$$(2) \quad \frac{T_f}{T_i} = 1 = n^0$$

In realistic process depending on friction we may realistically get n^{α} decrease with $0 < \alpha < 2/3 \Rightarrow \alpha = 1/2$ can be achieved