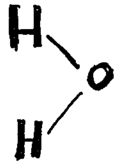
Quiz #1 3760.

Problem #1

Let's first estimate the temperature of the products coming out of the nozzle (you may actually skip this step)

• 1) all degrees of freedom are active 2) The temperature of products is very high => the equipartition theorem must work.



– molecule of water is not axial therefore we have 3 rotational degrees of freedom. So we have.

Translation 3 d. of. f.

Rotation 3 d. of. f.

Vibration $3\cdot 3 - 6 = 3N - 6 = 3$ d. of. f. (N – number of atoms in molecule)

Equipartition theorem: every transl. and rotational d. of. f carries $\frac{1}{2}kT$ amount of energy, every vibrational d. of. f. carries $kT = \underbrace{\frac{1}{2}kT}_{\text{potential energy of spring } \frac{1}{2}kx^2} + \underbrace{\frac{1}{2}kT}_{\text{kinetic energy } \frac{1}{2}mv^2}$ amount of energy

• The energy gained as a result of the chemical reaction will be split between different d. of. f.

(1) $$3\cdot\frac{1}{2}kT + 3\left(\frac{1}{2}kT\right) + 3\left(kT\right) = 483\,\frac{kJ}{N_a\cdot 2}$$

factor of 2 is because the equation is for 2 molecules of $H_2O$

$$6\,KT = \frac{483 \cdot 10^3}{6 \cdot 10^{23} \cdot 2} \qquad k = 1.38 \cdot 10^{-23}$$

$$T = \frac{483 \cdot 10^3}{8.3 \cdot 6 \cdot 2} \approx 4850\ K$$

[ (For comparison effective temperature of sun 5800k. temperature $1.5 \cdot 10^6$ k)
4488 k – record high melting temperature of $Ta_4HfC_5$
3695 k – melting point of tungsten
[illegible] are used in rocket nozzles, and also for body coverage of shuttles. ]

Velocity estimate.

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT \qquad \langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$ – that is why we have factor 3

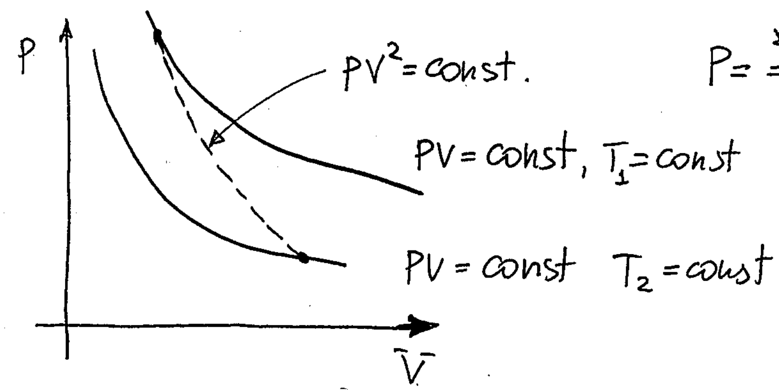
$$v^2 = \frac{3kT}{m} = \frac{3 \cdot k \cdot T}{\left(\frac{\mu}{N_A}\right)} = \frac{3 \cdot k \cdot N_a \cdot T}{\mu} = \frac{3 \cdot RT}{\mu}$$

$$= \frac{3 \cdot 8.3 \cdot 4850}{28 \cdot 10^{-3}} = 4.2 \cdot 10^6\ \frac{m^2}{s^2}$$

$v = 2000$ m/s.

(you may skip. T computation and get $\langle v \rangle$ from equation 1.

## Problem #2.

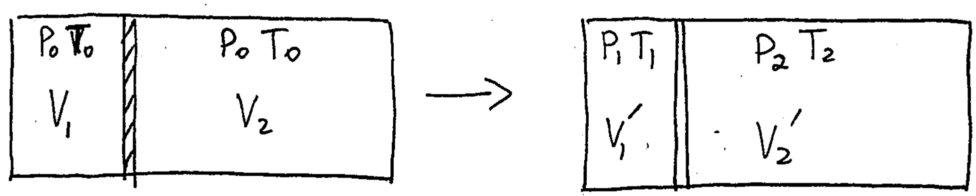



$$P = \frac{\text{const}}{V^2}$$

$$T_1 > T_2$$

Temperature decreases upon expansion with $PV^2$ = const law.

## Problem #3




$P_1 = P_2$ – mechanical equilibrium $\quad T_2 = 2T_1$.

Find $P_1$

From ideal gas law

eq (1) $$\frac{P_0 V_1}{T_0} = N_1 k = \frac{P_1 V_1'}{T_1} \Rightarrow \frac{P_0}{T_0} = \frac{P_1 V_1'}{V_1 \cdot T_1}$$

$$\frac{P_0 V_2}{T_0} = N_2 k = \frac{P_1 V_2'}{T_2} \Rightarrow \frac{P_0}{T_0} = \frac{P_1 V_2'}{V_2 \cdot T_2}$$

$$\left.\begin{aligned} \frac{P_1 V_1'}{V_1 T_1} &= \frac{P_1 V_2'}{V_2 T_2} \\ V_2 &= 2V_1 \\ T_2 &= 2T_1 \end{aligned}\right\} \Rightarrow V_1' = \frac{1}{4} V_2'$$

$V_1' + V_2' = V_1 + V_2$ ( volume is the same)

$5V_1' = 3V_1$

From eq. (1)

$$P_1 = \frac{T_1}{V_1'} \cdot \frac{P_0 V_1}{T_0} = \frac{5}{3} \frac{T_1}{T_0} \cdot P_0$$

$$\boxed{P_1 = \frac{5}{3} \frac{T_1}{T_0} \cdot P_0}$$