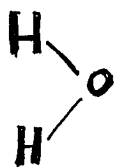


QUIZ #1 3760.

Problem #1

Let's first estimate the temperature of the products coming out of the nozzle (you may actually skip this step)

- 1) all degrees of freedom are active 2) The temperature of products is very high \Rightarrow the equipartition theorem must work.



- molecule of water is not axial therefore we have 3 rotational degrees of freedom. So we have.

Translation 3 d.o.f.

Rotation 3 d.o.f.

Vibration $3 \cdot 3 - 6 = 3N - 6 = 3$ d.o.f. (N - number of atoms in molecule)

Equipartition theorem: every transl. and rotational d.o.f. carrier $\frac{1}{2}kT$ amount of energy, every vibrational d.o.f. carrier $kT = \frac{1}{2}kT + \frac{1}{2}kT$ amount of energy

potential energy of spring $\frac{1}{2}kx^2$

kinetic energy $\frac{1}{2}mv^2$

- The energy gained as a result of the chemical reaction will be split between different d.o.f.

$$(1) \quad 3 \cdot \frac{1}{2}kT + 3 \left(\frac{1}{2}kT\right) + 3(kT) = 483 \frac{kT}{Na \cdot 2}$$

factor of 2 is because the equation is for 2 molecules of H_2O

(1)

$$6kT = \frac{483 \cdot 10^3}{6 \cdot 10^{23} \cdot 2}$$

$$k = 1.38 \cdot 10^{-23}$$

$$T = \frac{483 \cdot 10^3}{8.3 \cdot 6 \cdot 2} \approx 4850 \text{ K}$$

(For comparison effective temperature of sun 5800k
temperature $1.5 \cdot 10^6 \text{ K}$)

4488 K - record high melting temperature of Ta_4HfC_5

3695 K - melting point of tungsten

are used in rocket nozzles, and also for
body coverage of shuttles.

Velocity estimate.

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle - \text{that is why}$$

we have factor 3

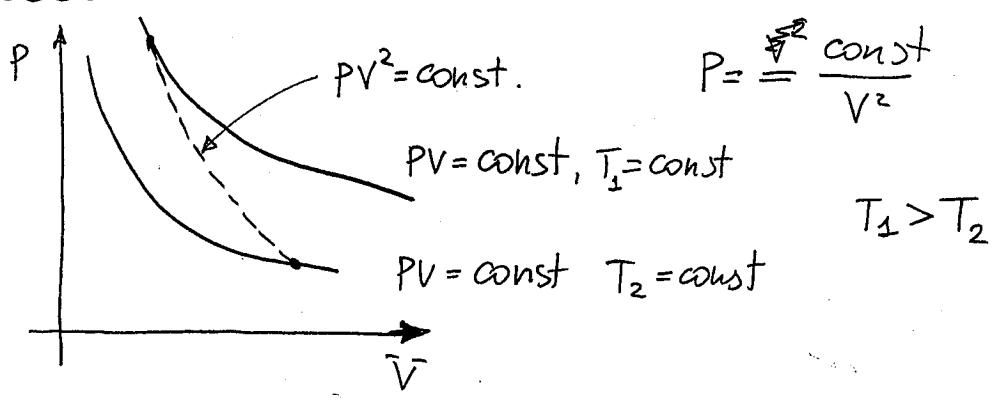
$$v^2 = \frac{3kT}{m} = \frac{3 \cdot k \cdot T}{\left(\frac{\mu}{N_A}\right)} = \frac{3 \cdot k \cdot N_A \cdot T}{\mu} = \frac{3 \cdot RT}{\mu}$$

$$= \frac{3 \cdot 8.3 \cdot 4850}{28 \cdot 10^{-3}} = 4.2 \cdot 10^6 \frac{\text{m}^2}{\text{s}^2}$$

$$v = 2000 \text{ m/s.}$$

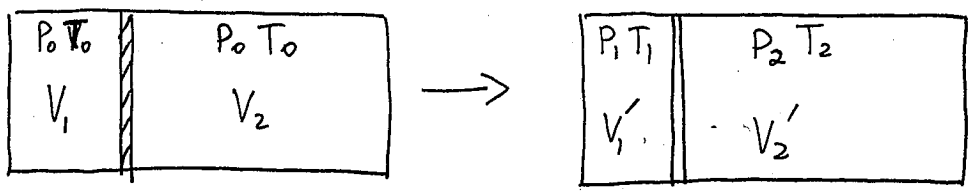
(you may skip T computation and get $\langle v \rangle$ from
equation 1.)

Problem #2.



Temperature decreases upon expansion with $PV^2 = \text{const}$ curve.

Problem #3



$P_1 = P_2$ - mechanical equilibrium $T_2 = 2T_1$.
Find P_1

From ideal gas law

$$\begin{aligned} \text{eq (1)} \quad \frac{P_0 V_1}{T_0} = N_2 k = \frac{P_1 V_1'}{T_1} &\Rightarrow \frac{P_0}{T_0} = \frac{P_1 V_1'}{V_1 \cdot T_1} \\ \frac{P_0 V_2}{T_0} = N_2 k = \frac{P_2 V_2'}{T_2} &\Rightarrow \frac{P_0}{T_0} = \frac{P_1 V_2'}{V_2 \cdot T_2} \\ \frac{P_1 V_1'}{V_1 T_1} = \frac{P_1 V_2'}{V_2 T_2} &\Rightarrow V_2 = 2V_1, \quad T_2 = 2T_1 \\ &\Rightarrow V_1' = \frac{1}{4} V_2' \end{aligned}$$

$$V_1' + V_2' = V_1 + V_2 \quad (\text{volume is the same})$$

$$5V_1' = 3V_1$$

From eq. (1)

$$P_1 = \frac{T_1}{V_1'} \cdot \frac{P_0 V_1}{T_0} = \frac{5}{3} \frac{T_1}{T_0} \cdot P_0$$

$$P_1 = \frac{5}{3} \frac{T_1}{T_0} \cdot P_0$$