Midterm 2

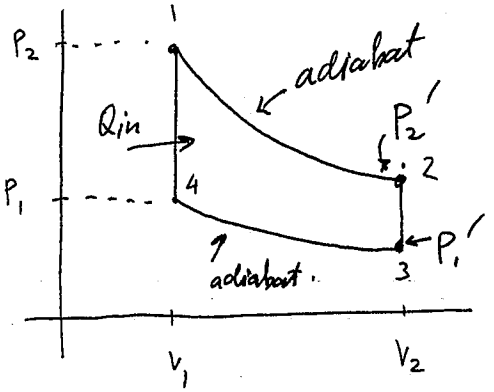
Problem 1

(a) $Z = \sum_i \exp\left(-\frac{\varepsilon_i}{kT}\right) = 1 + 2 \cdot \exp\left(-\frac{\varepsilon}{kT}\right)$

(b) $P(\varepsilon = 0) = \frac{\exp\left(-\frac{\varepsilon_0}{kT}\right)}{Z} = \frac{1}{1 + 2\exp\left(-\frac{\varepsilon}{kT}\right)}$

Midterm #2

Problem #2.



1) η -?

2) $\Delta S_{4\to 1}$?

3) $\Delta S_{1\to 2}$?

(a) $\eta = \frac{W}{Q_{in}}$

• heat goes into the system only at $4\to 1$ section of diagram

$\Delta W_{41} = 0 \Rightarrow Q_{in} = \Delta U_{14} = \frac{3}{2} nR\cdot(T_1 - T_4) = \frac{3}{2}(P_2V_1 - P_1V_1)$

W_{21} -? $\Delta Q_{21} = 0$ because the compression is adiabatic.

$\Rightarrow W_{21} = -\Delta U_{21} = \frac{3}{2} nR(T_1 - T_2) = \frac{3}{2}(P_2V_1 - P_2'V_2) =$

$= \frac{3}{2}\left(P_2V_1 - P_2\frac{V_1^{\gamma}}{V_2^{\gamma}}V_2\right)$ [for adiabat $P_2V_1^{\gamma} = P_2'V_2^{\gamma}$

$W_{34} = -\Delta U_{34} = \frac{3}{2}\left(P_1\frac{V_1^{\gamma}}{V_2^{\gamma}}V_2 - P_1V_1\right)$

$\eta = \frac{W_{21} + W_{34}}{\Delta U_{14}} = \frac{1}{V_1(P_2 - P_1)}\cdot\left[V_1(P_2 - P_1) - (P_2 - P_1)\frac{V_1^{\gamma}}{V_2^{\gamma}}V_2\right] =$

$= 1 - \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = 1 - \left(\frac{V_1}{V_2}\right)^{2/3}$

(b) ΔS_{41} $\quad dS = \frac{dQ}{T}$

$dQ = dU + dW$ $\quad$ ($dW = 0$ along $4 \to 1$ line because $\Delta V = 0$)

$dS = \frac{dU}{T}$

$$\Delta S = \int_{T_4}^{T_1} \frac{dU}{T} = \frac{3}{2} nR \cdot \int_{T_4}^{T_1} \frac{dT}{T} = \frac{3}{2} nR \ln\left(\frac{T_1}{T_4}\right) =$$

$$= \frac{3}{2} nR \cdot \ln\left(\frac{P_2 V_1}{P_1 V_1}\right) = \frac{3}{2} nR \ln\left(\frac{P_2}{P_1}\right)$$

(c) ΔS_{12} $\quad dQ_{12} = 0 \Rightarrow \Delta S_{12} = 0$. no entropy change.