Problem 1. [5 points]

In linear, isotropic dielectric the polarization is related to the electric field in the medium by the simple relation \( \mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \), where \( \chi_e \) is a constant called the dielectric susceptibility. Using the solution for the field of a uniformly polarized dielectric sphere, sketched on page 190 of the lecture notes, show that the field \( \mathbf{E} \) of a sphere of radius \( R \), made of linear dielectric material, and placed in the external uniform field \( \mathbf{E}_0 \) is given by the following expressions:

\[
\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 + \frac{\chi_e}{3} \frac{R^3}{r^3} \left[ 3 \hat{r} (\mathbf{E}_0 \cdot \hat{r}) - \mathbf{E}_0 \right]
\]

for \( r \) outside the sphere, and

\[
\mathbf{E}(\mathbf{r}) = \frac{\mathbf{E}_0}{1 + \chi_e/3}
\]

inside the sphere.

Problem 2. [5 points]

Point charge \( q \) is placed at the flat interface between two homogeneous dielectrics with dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \), correspondingly. The dielectrics are filling upper (lower) half-spaces out to spatial infinity. Find electric potential \( \phi(\mathbf{r}) \), electric field \( \mathbf{E}(\mathbf{r}) \) and displacement \( \mathbf{D}(\mathbf{r}) \).

Hint: similarly for the case of point charge in vacuum, look for a radially-symmetric solution for the electric field

\[
\mathbf{E}(\mathbf{r}) \sim \frac{r}{r^3}
\]

but with a different pre-factor.

Problem 3. [5 points] Consider a Drude metal placed in external homogeneous electric field \( \mathbf{E}_0 \cos \omega t \). Find the power dissipated per unit of volume \( P(\omega) \) and sketch it as the function of frequency. What is \( P(\omega) \) in the limit when mean free scattering time \( \tau \rightarrow 0 \)? Do it by
taking into account the charge build-up at the boundaries of the metal. Contrast the result with the dissipated power obtained when the charge build-up is neglected.

*Hints:* dissipated power is given by the (averaged over time) Joule heat $j \cdot E$, where $E$ is the *local* value of the electric field. Be careful with the quadratic quantities when utilizing complex functions. The notes from the class concerning Drude model will be posted on the course webpage.