Problem 1. [5 points]
Consider dielectric for which imaginary part of permittivity (dielectric function) is given by
\[ \varepsilon''(\omega) = \frac{\beta_0 \omega \tau}{1 + \omega^2 \tau^2}, \]
where \( \beta_0 \) and \( \tau \) are material constants. Use Kramers-Kronig relations to find the real part, \( \varepsilon'(\omega) \). Use this result to write complex \( \epsilon(\omega) \) in a compact form.

Problem 2. [10 points]
Consider the surface plasmon problem (the note on this topic, surface plasmon, a.k.a. surface plasmon-polariton, will be posted on the website): a metal, described by the dielectric function \( \varepsilon(\omega) = \varepsilon_0 (1 - \omega_0^2 / \omega^2) \), has a flat surface \( (z = 0 \text{ plane}) \) of contact with the vacuum above it \( (z > 0) \). Consider the wave localized near the surface:
\[ \mathbf{E} = (E_x(z), 0, E_z(z)) e^{i k x}, \]
where the \( x \)-component of electric field satisfies the Maxwell equations and the condition of continuity at the surface:

\[ E_x(z) = A \begin{cases} \exp \left( -z \sqrt{k^2 - \omega^2 / c^2} \right), & z > 0, \\ \exp \left( z \sqrt{k^2 - \varepsilon \omega^2 / \varepsilon_0 c^2} \right), & z < 0. \end{cases} \]

Find all components of the magnetic field \( \mathbf{B}(x, z) \) in the metal and in the vacuum (express them via the amplitude \( A \)) and verify that they are continuous at the surface. Based on your results, is the surface plasmon a EM wave, a TM wave or neither one of them?

For the reference, the Maxwell equations for the fields oscillating with time as \( \propto e^{-i \omega t} \) in the absence of free charges (and magnetization) are
\[ \nabla \times \mathbf{E} = i \omega \mathbf{B}, \quad \nabla \times \mathbf{B} = -i \omega \frac{\varepsilon(\omega)}{\varepsilon_0} \mathbf{E}. \]

Problem 3. [5 points]
Continuing with the setup of the previous problem, consider now the case of a wave polarized along the \( y \)-direction:
\[ \mathbf{E} = (0, E_y(z), 0) e^{i k x}. \]
From the Maxwell equations find $E_y(z)$ (up to a constant amplitude) and all the components of the magnetic field (both in the metal and in the vacuum) and make sure all the boundary conditions (continuity of $E_x$, $E_y$, $D_z$ and $B$ at the surface) are satisfied. Can such a surface wave exist?

**Problem 4.** [6 points]

Find electric current $\vec{j}(t)$ in an infinite conductor in response to a sudden switching of a homogeneous electric field $\vec{E}(t) = \Theta(t)\vec{E}_0$, where $\Theta(t)$ is Heavyside step function.

**Problem 5.** [4 points]

Solve Problem 4 assuming now that the conductor has finite length in the direction of the applied electric field. Use discussion note from Tuesday, March 1 class. Sketch the $t$-dependence of the obtained results for $j(t)$ from Problems 4 and 5 on the same plot.