Physics 7910: HW # 01.

(Dated: January 17, 2013)

Homework is due January 31, 2013.

1. Using commutator relations for the raising and lowering angular momentum operators $J_\pm = J_x \pm iJ_y$, $[J_+, J_-] = 2J_z$ and $[J_z, J_\pm] = \pm J_\pm$, deduce the action of these operators on an angular momentum eigenstate $|j, m\rangle$. (This is the standard state that satisfies $\mathbf{J}^2|j, m\rangle = \hbar^2 j(j+1)|j, m\rangle$ and $J_z|j, m\rangle = \hbar m|j, m\rangle$.)

2. Show that the operator $S_\mathbf{n} = \sin\theta\cos\phi S_x + \sin\theta\sin\phi S_y + \cos\theta S_z$, which represents the spin-1/2 operator for the component of spin along a direction specified by the unit vector $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, has eigenvalues $\pm\hbar/2$. Find its eigenstates. Show also that $S_\mathbf{n}^2 = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Here $S_a = \frac{\hbar}{2}\sigma_a$ and σ_a are the Pauli matrices.

3. Hamiltonian of a spin in magnetic field along the z-axis is given by $H = \frac{g\mu_B}{\hbar}\vec{B}\cdot\vec{S} = \frac{g\mu_B}{\hbar}BS_z$. Use equation of motion for the expectation value of the spin, $i\hbar\frac{d}{dt}\langle\vec{S}\rangle = \langle[\vec{S}, H]\rangle$ to derive spin precession $i\hbar\frac{d}{dt}\langle\vec{S}\rangle == -ig\mu_B\langle\vec{S}\rangle \times \vec{B}$.

4. Using rules for addition of angular momenta, construct all eigenstates $|j, m\rangle$ that obtain from adding angular momentum $J_1 = 1$ and angular momentum $J_2 = 1/2$. Here, as usual, $\mathbf{J}^2|j, m\rangle = \hbar^2 j(j+1)|j, m\rangle$ and $J_z|j, m\rangle = \hbar m|j, m\rangle$, and $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$ is the operator of the total angular momentum while $J_z = J_{1z} + J_{2z}$ is its z-component.