Emergent Ising orders of frustrated magnets

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Many of the materials are quantum magnets (e.g. linarite) and many more of them are from Utah :)

Musée des Minéraux, UPMC
Interacting magnons

- Repulsive:
  - superfluid

- Attractive:
  - collapse

Anything else?

-> emergence of composite order = nematic
Exotic ordered phases, emergent (Ising) orders

ordered phases

spin nematic

composite order parameter

\( O^{\alpha\beta}(r_i, r_j) = \frac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \frac{1}{3}\delta^{\alpha\beta}\langle S_i \cdot S_j \rangle \)

quantum spin liquids

spin nematic

composite order parameter

Spin nematics

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(Submitted 6 March 1984)


We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid \(^4\text{He}\) is discussed.
Emergent Ising order parameters

Chiral spin liquid in two-dimensional XY helimagnets
A. O. Sorokin and A. V. Syromyatnikov

\[ H = \sum_x (J_1 \cos(x - x_{+a}) + J_2 \cos(x - x_{+2a}) - J_4 \cos(x - x_{+4a})) \]

Two successive transitions were observed with the temperature decreasing: the first one is associated with breaking of a discrete symmetry and the second one is of the Berezinskii-Kosterlitz-Thouless type.

Two-step restoration of SU(2) symmetry in a frustrated ring-exchange magnet
A. Läuchli, J. C. Domenge, C. Lhuillier, P. Sándorg, and M. Troyer

Twelve sublattice ordered phase in the J_1-J_2 model on the kagomé lattice
J.-C. Domenge, P. Sándorg, and C. Lhuillier

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets
Luca Capriotti and Subir Sachdev

\[ H = J_1 \sum_{\langle i,j \rangle} \hat{S}_i \cdot \hat{S}_j + J_2 \sum_{\langle \langle i,j \rangle \rangle} \hat{S}_i \cdot \hat{S}_j \]

\[ \sigma_a = (\hat{S}_1 \cdot \hat{S}_3 - \hat{S}_2 \cdot \hat{S}_3)_a \]

FIG. 2. The two different minimum energy configurations with magnetic wave vectors \( Q = (Q, Q) \) and \( Q^* = (Q, -Q) \) with \( Q = 2\pi/3 \), corresponding to \( J_2/J_1 = 0.5 \).

Motivated by recent experiments on an S=1/2 antiferromagnet on the kagomé lattice, we investigate the Heisenberg-J_1-J_2 model with ferrimagnetic and antiferromagnetic J_2. Classically the ground state displays Néel long-range order with 12 noncollinear sublattices. The order parameter has the symmetry of a cuboctahedron, it breaks SU(3) as well as the SU(2)-symmetry, and we expect from the Néel-Wigner theorem in two dimensions, the 2H3 symmetry is restored by thermal fluctuations while the J2 symmetry breaking persists up to a finite temperature. A complete study of S=1/2 exact spectra reveals that the classical order sublattices for quantum spins is a finite range of parameters. First-order spin wave equations give rise to the range of this phase and these modifications at T=0 of the order parameters associated to both breaking phases. This paper is motivated by quantum fluctuations for small but finite J2/J1=3, consistent with exact spectra, which indicate a gapped phase.
Outline

* Magnetization process of triangular lattice antiferromagnet
  - accidental degeneracy and selection of the magnetization plateau at \( M = 1/3 \) by quantum (thermal) fluctuations

* Instabilities of the plateau
  - moving down/up in magnetic field
  - spatial anisotropy of exchange interactions

* Two-magnon pairing and condensation
  - Spin-current (chiral Mott) phase
  - Spontaneous generation of DM interaction!

* Generic features: (at least) two-fold spectrum degeneracy
  strong interactions (in the vicinity of single-particle condensation)

• Mott (PM) to Superfluid (cone) transition
  proceeds via intermediate chiral Mott phase
  \( U(1) \times Z_2 \)
Classical isotropic triangular AFM in magnetic field

- Zero field: co-planar spiral (120 degree) state
- Magnetic field: accidental degeneracy
  \[ H = J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{h} \cdot \vec{S}_i \]
  \[ H = \frac{1}{2} J \sum_{\triangle} \left( \sum_{i \in \triangle} \vec{S}_i - \frac{\vec{h}}{3J} \right)^2 \]
- all states with \( \vec{S}_{i1} + \vec{S}_{i2} + \vec{S}_{i3} = \frac{\vec{h}}{3J} \) form the lowest-energy manifold
- Accidental degeneracy
  - O(2) spins: 3 angles, 2 equations => 1 continuous angle undetermined
  - O(3) spins: 6 angles, 3 equations => 2 continuous angles (upto global U(1) rotation about \( \vec{h} \))

Order-by-disorder: This degeneracy is lifted by thermal/quantum fluctuations.
Phase diagram of the Heisenberg (XXX) model in the field

Phase Transition of the Two-Dimensional Heisenberg Antiferromagnet on the Triangular Lattice

Hikaru KAWAMURA and Seiji MIYASHITA

Phase Transition of the Heisenberg Antiferromagnet on the Triangular Lattice in a Magnetic Field

Hikaru KAWAMURA and Seiji MIYASHITA

Z\textsubscript{2} vortex (chirality ordering) transition

Seabra, Momoi, Sindzingre, Shannon 2011

Gvozdikova, Melchy, Zhitomirsky 2010
Quantum fluctuations, $S \gg 1$, $T=0$.

$J' = J$: Quantum fluctuations select co-planar and collinear phases

**UUD plateau is due to interactions between spin waves**

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

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Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and XY quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.

Figure 1. Reorientation process in the magnetic field in 2D Heisenberg AFM on a triangular lattice. Zero-point fluctuations stabilize the collinear phase in the finite region $H_1 < H < H_1$ in the vicinity of $H_{sat}/3$.

Figure 3. The anticipated behaviour of longitudinal magnetization in 2D Heisenberg AFM on a triangular lattice. The plateau on the magnetization curve results from the stabilization of the collinear phase in the finite region of magnetic fields due to zero-point motion.

$h_{c2} - h_{c1} = (0.6/2S) h_{sat}$

up-up-down collinear state
**Exp:** $M=1/3$ magnetization plateau in $\text{Cs}_2\text{CuBr}_4$

- First observation of “up-up-down” state in spin-1/2 triangular lattice antiferromagnet
- Total of 9 phases -- instead of 3 expected!

**Important:** the lattice is strongly anisotropic
Emergent Ising order near the end-point of the 1/3 magnetization plateau

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]

Spatially anisotropic model: classical vs quantum

\[ H = \sum_{\langle ij \rangle} J_{ij} S_i \cdot S_j - \hbar \sum_i S_i^z \]

Umbrella state: favored classically; energy gain \((J - J')^2 / J\)

Planar states: favored by quantum fluctuations; energy gain \(J / S\)

The competition is controlled by dimensionless parameter \(\delta = S(J - J')^2 / J^2\)

- Umbrella state: \(S = \infty\), \(J' \neq J\)
- Planar states: \(S = 1/2\)
- 1/3-plateau

The competition is controlled by dimensionless parameter \(\delta = S(J - J')^2 / J^2\)
Notable exception: spin-\(1/2\) triangular lattice AFM is different

Ground states of spin-\(1/2\) triangular antiferromagnets in a magnetic field

\[ h/J \]

\[ R=1-J'/J \]

S=1/2 vs S=1 difference for the ordered 2d phase!!!
Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at $T=0$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$
Emergent Ising order near the end-point of the 1/3 magnetization plateau — the result

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$

OAS, Reports on Progress in Physics 78, 052502 (2015),
UUD-to-cone phase transition

$Z_3 \rightarrow U(1) \times Z_2$ or $Z_3 \rightarrow \text{smth else} \rightarrow U(1) \times Z_2$?

$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$

$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$
Low-energy excitation spectra

\[ \varepsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4} \]

for \( \delta < 3 \)

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete \( Z_3 \). Hence, very stable.

\[ h_{c2} - h_{c1} = \frac{0.6}{2S} h_{sat} = \frac{0.6}{2S}(9JS) \]

Bose-Einstein condensation of \( d_1(d_2) \) mode at \( k = 0 \) leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009
Low-energy excitation spectra near the plateau’s end-point

\[ \delta = \frac{40}{3} \frac{S}{J'} (1 - J'/J)^2 \] parameterizes anisotropy \( J'/J \)

Magnetization plateau is \textbf{collinear} phase: preserves 
\( O(2) \) rotations about magnetic field -- 
no gapless spin waves. 
Breaks only discrete \( Z_3 \).
Interaction between low-energy magnons

\[ \mathcal{H}_{d_1d_2}^{(4)} = \frac{3}{N} \sum_{p,q} \Phi(p, q) \left( d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger d_{1,-k_0+q} d_{2,k_0-q} - d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger d_{1,-k_0+q} d_{2,k_0-q} \right) + \text{h.c.} \]

\[ \Phi(p, q) \sim \frac{(-3J)k_0^2}{|p||q|} \]

singlular magnon interaction

magnon pair operators

\[ \Psi_{1,p} = d_{1,k_0+p} d_{2,-k_0-p} \]
\[ \Psi_{2,p} = d_{1,-k_0+p} d_{2,k_0-p} \]

Obey canonical Bose commutation relations in the UUD ground state

\[ [\Psi_{1,p}, \Psi_{2,q}] = \delta_{1,2} \delta_{p,q} \left( 1 + d_{1,k_0+p}^\dagger d_{1,k_0+p} + d_{2,k_0+p}^\dagger d_{2,k_0+p} \right) \rightarrow \delta_{1,2} \delta_{p,q} \]

In the UUD ground state \[ \langle d_{1}^\dagger d_{1} \rangle_{uud} = \langle d_{2}^\dagger d_{2} \rangle_{uud} = 0 \]

★ Interacting magnon Hamiltonian in terms of \( d_{1,2} \) bosons = non-interacting Hamiltonian in terms of \( \Psi_{1,2} \) magnon pairs

Chubukov, OS PRL 2013
Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for $\Psi$ - Hamiltonian

\[
\langle \Psi_{1,p}^\dagger - \Psi_{1,p} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f^2_q \langle \Psi_{2,q}^\dagger - \Psi_{2,q} \rangle
\]

\[
\langle \Psi_{2,p}^\dagger - \Psi_{2,p} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f^2_q \langle \Psi_{1,q}^\dagger - \Psi_{1,q} \rangle
\]

`Superconducting' solution with *imaginary* order parameter

\[
\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{p^2}
\]

Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

\[
1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1 - \delta/4)k_0^2}}
\]

no single particle condensate

\[
\langle d_1 \rangle = \langle d_2 \rangle = 0
\]
Two-magnon condensate = Spin-current nematic state

\[ h \hat{z} \cdot S_A \times S_C = h \hat{z} \cdot S_C \times S_B = h \hat{z} \cdot S_B \times S_A \propto \gamma \]

Finite scalar (and vector) chiralities. Sign of \( \gamma \) determines sense of spin-current circulation

Spontaneously broken \( \mathbb{Z}_2 \) -- spatial inversion [in addition to broken \( \mathbb{Z}_3 \)
inhired from the UUD state]

Leads to spontaneous generation of Dzyaloshinskii-Moriya interaction

Chubukov, OS PRL 2013
Spontaneous generation of Dzyaloshinskii-Moriya interaction

\[ \mathcal{H}^{(4)}_{d_1 d_2} \propto \frac{1}{N} \sum_{k \in +k_0} \frac{-i k_0}{\sqrt{(k - k_0)^2 + (1 - \delta/4)k_0^2}} (d^\dagger_{1,k} d^\dagger_{2,-k} - d_{1,k} d_{2,-k}) \sum_{p \in -k_0} \frac{-i k_0}{\sqrt{(p + k_0)^2 + (1 - \delta/4)k_0^2}} (d^\dagger_{1,p} d^\dagger_{2,-p} - d_{1,p} d_{2,-p}) \]

continuum limit of DM in triangular lattice

\[ \sum_{\mathbf{r}} \hat{z} \cdot \mathbf{S}_\mathbf{r} \times (\mathbf{S}_{\mathbf{r}+\mathbf{a}_1} + \mathbf{S}_{\mathbf{r}+\mathbf{a}_2}) \quad \mathbf{B} \parallel \mathbf{z} \]

Mean-field approximation:

\[ \mathcal{H}^{(4)}_{d_1 d_2} \rightarrow D \sum_k \left( \frac{k_0}{|k - k_0|} + \frac{k_0}{|k + k_0|} \right) (d^\dagger_{1,k} d^\dagger_{2,-k} - d_{1,k} d_{2,-k}) \]

\[ D \sim \Upsilon \]

spin currents appear due to \textbf{spontaneously generated DM}

Precessing spins on sub lattices A, B, C are phase shifted by $2\pi/3$:

$$
\mathbf{S}_A = (\cos[\omega t], \sin[\omega t], M_A), \quad \mathbf{S}_B = (\cos[\omega t \pm \frac{4\pi}{3}], \sin[\omega t \pm \frac{4\pi}{3}], M_B), \quad \mathbf{S}_C = (\cos[\omega t \pm \frac{2\pi}{3}], \sin[\omega t \pm \frac{2\pi}{3}], M_C)
$$

Then no dipolar transverse order:

$$
\langle \mathbf{S}_r^{x,y} \rangle = 0 \quad \text{and} \quad \langle \mathbf{S}_A \cdot \mathbf{S}_C \rangle = \langle \mathbf{S}_C \cdot \mathbf{S}_B \rangle = \langle \mathbf{S}_B \cdot \mathbf{S}_A \rangle = \cos[\frac{2\pi}{3}]
$$

But finite chirality, determined by the sign of $2\pi/3$ shift between the sublattices:

$$
\langle \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \mathbf{S}_B \times \mathbf{S}_A \rangle = \pm \sin[\frac{2\pi}{3}]
$$
Continuous transitions: plateau $\rightarrow$ spin-current $\rightarrow$ cone !

\[ Z_3 \rightarrow Z_3 \times Z_2 \rightarrow U(1) \times Z_2 \]

\[
H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

\[
\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2
\]
Spin-current phase = chiral Mott insulator

gapped single particles;
but spontaneously broken time-reversal
= spontaneous circulating currents
Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

- Spin-1 model with featureless Mott ground state at large $D > 0 \ [S^z_r = 0]$
- Triangular lattice: two-fold degenerate spectrum, at $+Q$ and $-Q$

\[
H = \sum_{\langle r, r' \rangle} J(S^x_r S^x_{r'} + S^y_r S^y_{r'} + \Delta S^z_r S^z_{r'}) + D \sum_r (S^z_r)^2
\]

1. Toy problem of two-spin exciton. Derive Schrödinger eqn for the pair wave function $\psi$

\[
|\text{ex}\rangle = \sum_{n \neq m} \psi_{n,m} |n, m\rangle \text{ where } |n, m\rangle = \frac{1}{2} S^+_n S^-_m |0\rangle_j
\]

Solution which is **odd** under inversion

- is the first instability when approaching from large-$D$ limit.
- Indicates chiral Mott phase.

[Single-particle condensation occurs at $D=3J$.]
Schwinger boson representation of $S=1$

\[
S_r^z = b_r^\dagger S^z b_r = b_{r\uparrow}\dagger b_{r\uparrow} - b_{r\downarrow}\dagger b_{r\downarrow},
\]
\[
S_r^+ = b_r^\dagger S^+ b_r = \sqrt{2}(b_{r\uparrow}\dagger b_{r0} + b_{r0}\dagger b_{r\downarrow}),
\]
\[
S_r^- = b_r^\dagger S^- b_r = \sqrt{2}(b_{r\downarrow}\dagger b_{r0} + b_{r0}\dagger b_{r\uparrow}).
\]

Large-D limit: $b_0$ is condensed, $b_{\uparrow,\downarrow}$ are excitations about the vacuum.

Effective interaction is made of a number of 2nd order processes

+ many more…
Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

Spin-1 model with featureless Mott ground state at large $D > 0$

Triangular lattice

$$H = \sum_{\langle r,r' \rangle} J (S^x_r S^x_{r'} + S^y_r S^y_{r'} + \Delta S^z_r S^z_{r'}) + D \sum_r (S^z_r)^2$$

2. Solve Bethe-Salpeter eqn.

FIG. 2: The two particle propagator is obtained as a sum of ladder diagrams.

Preliminary result

$\Delta$
Conclusions

**Mott -> superfluid** transition in frustrated lattice requires U(1) x Z₂ breaking.

This proceeds via intermediate **spin-current** (chiral Mott) phase (breaking Z₂ only).

Spontaneously breaks spatial inversion.

All single particle excitations are gapped.

Thank you!
The end-plateau 2-magnon instability occurs at \( \delta \sim 1/\sqrt{S} \gg 1/S^2 \)

Should be checked by numerics!

Wen Jin, OAS
Magnetic phase transitions are detected electrically!

Magnetoresistivity and Monte Carlo studies of magnetic phase transitions in C₆Eu

Phys. Rev. B 34, 423 – Published 1 July 1986
Other systems

Ferrimagnetic state of kagome (anti)ferromagnet volborthite
@ 1/3 magnetization plateau

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite $\text{Cu}_3\text{V}_2\text{O}_7(\text{OH})_2\cdot 2\text{H}_2\text{O}$

$J_1 < 0$, $J_2 > 0$, $J' > 0$

J$_1$ FM, J$_2$ AF

J' AF

polarized chains?!
Frustrated ferromagnetism

PHYSICAL REVIEW B 82, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite Cu$_3$V$_2$O$_7$(OH)$_2$$\cdot$2H$_2$O

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DFT gets it right!

$J_1 < 0, J_2 > 0, J' > 0$
Other systems

Semiclassical analysis of a magnetization plateau in a 2D frustrated ferrimagnet

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arXiv:1610.03135

Kagome geometry

Spin-current pattern

FIG. 1. Proposed Enantiomers for vilbushite. The blue dots represent spin-1/2 copper ions and the line segments represent Heisenberg couplings. $J_1 < 0$ is ferromagnetic while $J_2 > 0$ and $J' > 0$ are antiferromagnetic. The distances between adjacent unit cells is slightly anisotropic, with $a = 5.94$ Å and $a' = 6.07$ Å [16]. Capital letters label the sublattices.

FIG. 6. Schematic quantum phase diagram at $J' = 0.5|J_1|$. The UUD state breaks no symmetries, the gapped chiral liquid (CL) phase only breaks chiral symmetry, and the gapless cone state breaks both chiral and a $U(1)$ symmetry combining translation and spin rotation. The thick solid lines and dashed lines represent first- and second-order transitions respectively. We did not investigate the nature of the transition between the chiral liquid and cone phases represented by the red line. In a 3D phase diagram like that of Fig. 4 that includes the applied field, the chiral liquid phase would appear as a thin tube around the stabilization curve where the two sheets meet.

FIG. 7. Spin-current configuration in the chiral liquid phase. The magenta arrows indicating the direction of spin current flow; in the orthogonal ground state the flow is reversed. The spin current on the diagonal bonds, represented by thicker arrows, is larger by a factor of $\sqrt{2}$ and determines the net current flow. The ground state is chiral and spontaneously breaks the lattice symmetry of reflection about the dotted line.