Emergent Ising orders of frustrated magnets

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Quantum magnetism vs Spintronics

Spin is (almost) conserved

- No dipolar coupling (small magnetic moments)
  Notable exception —> spin ice

- Basically isolated system of spins
- No coupling to phonons, etc.

Notable exception -> hybridization of magnons and phonons in non-collinear spin structures

- Spin transport —> mostly thermal
  1) heat transport in chains/ladders
  2) thermal transport in organic spin liquid candidate materials
  3) magnon Hall effect (due to DM interactions)
  4) Spin Seebeck effect in ordered and critical magnetic insulators
Interacting magnons

- Repulsive
  - superfluid
- Attractive
  - collapse

Anything else?

-> emergence of composite order = nematic
Exotic ordered phases, emergent (Ising) orders

ordered phases
spin nematic
composite order parameter
quantum spin liquids

\[ \mathcal{O}^{\alpha\beta}(r_i, r_j) = \frac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \frac{1}{3}\delta^{\alpha\beta}\langle S_i \cdot S_j \rangle \]

A MAGNETIC ANALOGUE OF STEREOISOMERISM: APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

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Spin nematics

A. F. Andreev and I. A. Grishchuk

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(Submitted 6 March 1984)


We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid \(^7\)He is discussed.
Emergent Ising order parameters

Chiral spin liquid in two-dimensional $XY$ helimagnets

A. O. Sorokin$^{1, *}$ and A. V. Syromyatnikov$^{1,2,3}$

\[ H = \sum_{x} (J_1 \cos(\varphi_x - \varphi_{x+a}) + J_2 \cos(\varphi_x - \varphi_{x+2a}) - J_b \cos(\varphi_x - \varphi_{x+b})). \]

Low-Temperature Broken-Symmetry Phases of Spiral Antiferromagnets

Luca Capriotti$^{1,2}$ and Subir Sachdev$^{2,3}$

\[ \hat{H} = J_1 \sum_{\langle i,j \rangle} \hat{S}_i \cdot \hat{S}_j + J_3 \sum_{\langle i,j \rangle} \hat{S}_i \cdot \hat{S}_j, \]

\[ \sigma_a = (\hat{S}_1 \cdot \hat{S}_2 - \hat{S}_2 \cdot \hat{S}_3)_a, \]

FIG. 2. The two different minimum energy configurations with magnetic wave vectors $Q = (Q, Q)$ and $Q' = (Q, -Q)$ with $Q = 2\pi/3$, corresponding to $J_3/J_1 = 0.5$. 
Magnetization process of triangular lattice antiferromagnet
- accidental degeneracy and selection of the magnetization plateau at $M=1/3$ by quantum (thermal) fluctuations

Instabilities of the plateau
- moving down/up in magnetic field
- spatial anisotropy of exchange interactions

Two-magnon pairing and condensation
- Spin-current (chiral Mott) phase
- Spontaneous generation of DM interaction!

Generic features: (at least) two-fold spectrum degeneracy
strong interactions (in the vicinity of single-particle condensation)

Mott (PM) to Superfluid (cone) transition
proceeds via intermediate chiral Mott phase
$U(1) \times Z_2$
Phase diagram of the Heisenberg (XXX) model in the field

Phase Transition of the Two-Dimensional Heisenberg Antiferromagnet on the Triangular Lattice
Hikaru Kawamura and Seiji Miyashita

Phase Transition of the Heisenberg Antiferromagnet on the Triangular Lattice in a Magnetic Field
Hikaru Kawamura and Seiji Miyashita

Z\textsubscript{2} vortex (chirality ordering) transition

Seabra, Momoi, Sindzingre, Shannon 2011

Gvozdikova, Melchy, Zhitomirsky 2010
Quantum fluctuations, $S >> 1$, $T=0$.

$J' = J$: Quantum fluctuations select co-planar and collinear phases

**UUD plateau is due to interactions between spin waves**

Quantum theory of an antiferromagnet on a triangular lattice in a magnetic field

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Received 9 February 1990

Abstract. The reorientation process in a magnetic field in two-dimensional isotropic and $XY$ quantum Heisenberg antiferromagnets is shown to occur through the intermediate phase with unbroken continuous symmetry and constant magnetization equal to one third of the saturation value. The same reorientation process is also found in the more complicated classical models.

$h_{c2} - h_{c1} = (0.6/2S) h_{sat}$

up-up-down collinear state
**Exp:** $M=1/3$ magnetization plateau in Cs$_2$CuBr$_4$

★ Observed in Cs$_2$CuBr$_4$ (Ono 2004, Tsuji 2007)

\[ S=1/2 \]

★ first observation of “up-up-down” state in spin-1/2 triangular lattice antiferromagnet

★ total of 9 phases -- instead of 3 expected!

**Important:** the lattice is strongly anisotropic
Magnetic phase transitions are detected electrically!

Magnetoresistivity and Monte Carlo studies of magnetic phase transitions in $\text{C}_6\text{Eu}$

Phys. Rev. B 34, 423 – Published 1 July 1986
Emergent Ising order near the end-point of the 1/3 magnetization plateau

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]

OAS, Reports on Progress in Physics 78, 052502 (2015),
Spatially anisotropic model: classical vs quantum

$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i S_i^z$$

Umbrella state: favored classically; energy gain $(J-J')^2/J$

Planar states: favored by quantum fluctuations; energy gain $J/S$

The competition is controlled by dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$
Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at $T=0$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$
Emergent Ising order near the end-point of the 1/3 magnetization plateau — the result

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]

UUD-to-cone phase transition

\[ Z_3 \rightarrow U(1) \times Z_2 \text{ or } Z_3 \rightarrow \text{smth else} \rightarrow U(1) \times Z_2? \]

\[ H = \sum_{\langle i, j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]
Low-energy excitation spectra

\[ \epsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4} \]

for \( \delta < 3 \)

\[ \epsilon_{d1} = h - h_{c1} + \frac{3Jk^2}{4} \]

for \( \delta < 1 \)

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves.

Breaks only discrete \( Z_3 \).

Hence, very stable.

\[ h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\text{sat}} = \frac{0.6}{2S} (9JS) \]

Bose-Einstein condensation of \( d_1 \) (\( d_2 \)) mode at \( k = 0 \) leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009
Low-energy excitation spectra near the plateau’s end-point

\[ \delta = \frac{40S}{3}(1 - J'/J)^2 \] parameterizes anisotropy \( J'/J \)

extended symmetry: 4 gapless modes at the plateau’s end-point

\[ k_0 = \sqrt{\frac{3}{10S}} \]

S\( \gg 1 \)

Magnetization plateau is \textit{collinear} phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z\(_3\).

Alicea, Chubukov, OS PRL 2009
Interaction between low-energy magnons

\[ \mathcal{H}^{(4)}_{d_1d_2} = \frac{3}{N} \sum_{p,q} \Phi(p, q) \left( d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger d_{1,-k_0+q} d_{2,k_0-q} - d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger d_{1,-k_0+q}^\dagger d_{2,k_0-q} \right) + \text{h.c.} \]

\[ \Phi(p, q) \sim \frac{(-3J)k_0^2}{|p||q|} \]

singular magnon interaction

magnon pair operators

\[ \{ \Psi_{1,p} = d_{1,k_0+p} d_{2,-k_0-p}, \Psi_{2,p} = d_{1,-k_0+p} d_{2,k_0-p} \} \]

Obey canonical Bose commutation relations in the UUD ground state

\[ [\Psi_{1,p}, \Psi_{2,q}] = \delta_{1,2} \delta_{p,q} \left( 1 + d_{1,k_0+p}^\dagger d_{1,k_0+p} + d_{2,k_0+p}^\dagger d_{2,k_0+p} \right) \rightarrow \delta_{1,2} \delta_{p,q} \]

In the UUD ground state \[ \langle d_{1}^\dagger d_{1} \rangle_{\text{uud}} = \langle d_{2}^\dagger d_{2} \rangle_{\text{uud}} = 0 \]

★ Interacting magnon Hamiltonian in terms of \( d_{1,2} \) bosons = non-interacting Hamiltonian in terms of \( \Psi_{1,2} \) magnon pairs

Chubukov, OS PRL 2013
Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense before single magnons $d_{1,2}$

Equations of motion for $\Psi$ - Hamiltonian

$$\langle \Psi_{1,p}^\dagger - \Psi_{1,p} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi_{2,q}^\dagger - \Psi_{2,q} \rangle$$

$$\langle \Psi_{2,p}^\dagger - \Psi_{2,p} \rangle = \frac{6Jf_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi_{1,q}^\dagger - \Psi_{1,q} \rangle$$

`Superconducting’ solution with imaginary order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\gamma}{p^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|p|^2 + (1 - \delta/4)k_0^2}}$$

no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$
Two-magnon condensate = Spin-current nematic state

\[ \hat{h} \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C = \hat{h} \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B = \hat{h} \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \propto \Upsilon \]

Finite scalar (and vector) chiralities. Sign of \( \Upsilon \) determines sense of spin-current circulation

\( \langle \hat{S}^{x,y}_r \rangle = 0 \quad \langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle \) is not affected

Spontaneously broken \( Z_2 \) -- spatial inversion [in addition to broken \( Z_3 \) inherited from the UUD state]

Leads to spontaneous generation of Dzyaloshinskii-Moriya interaction

Chubukov, OS PRL 2013
The end-plateau 2-magnon instability occurs at $\delta \sim 1/\sqrt{S} >> 1/S^2$

Should be checked by numerics!

Wen Jin, OAS
Spontaneous generation of Dzyaloshinskii-Moriya interaction

$$\mathcal{H}_{d_1d_2}^{(4)} \propto \frac{1}{N} \sum_{k \in +k_0} \frac{-ik_0}{\sqrt{(k-k_0)^2 + (1-\delta/4)k_0^2}} (d_{1,k}^\dagger d_{2,-k}^\dagger - d_{1,k} d_{2,-k}) \sum_{p \in -k_0} \frac{-ik_0}{\sqrt{(p+k_0)^2 + (1-\delta/4)k_0^2}} (d_{1,p}^\dagger d_{2,-p}^\dagger - d_{1,p} d_{2,-p})$$

$$\mathcal{H}_{d_1d_2}^{(4)} \rightarrow D \sum_k \left( \frac{k_0}{|k-k_0|} + \frac{k_0}{|k+k_0|} \right) (d_{1,k}^\dagger d_{2,-k}^\dagger - d_{1,k} d_{2,-k})$$

$$D \sim \gamma$$

spin currents appear due to spontaneously generated DM

similar to Lauchli et al (PRL 2005) for Heisenberg+ring exchange model;
also ‘chiral Mott insulator’, Dhar et al, PRB 2013; Zaletel et al, 2013
Continuous transitions: plateau $\rightarrow$ spin-current $\rightarrow$ cone!

$$Z_3 \rightarrow Z_3 \times Z_2 \rightarrow U(1) \times Z_2$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$
Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan, Alexander O. Gogolin, and Fabian H. L. Essler

FIG. 3. Structure of the spin currents in the spin nematic phase.

Spin-current phase = chiral Mott insulator

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar, Tapan Mishra, Maheswar Maji, R. V. Pai, Subroto Mukerjee, and Arun Paramekanti

FIG. 1. Bosons on the Frustrated Triangular Lattice. (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion ξ_k, with minima at the K, K' points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution ⟨ˆn_k⟩ for the chiral phases.

Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel, S. A. Parameswaran, Andreas Rüegg, and Ehud Altman

gapped single particles; but
spontaneously broken time-reversal = spontaneous circulating currents
Other systems
Ferrimagnetic state of kagome (anti)ferromagnet volborthite
@ 1/3 magnetization plateau

\[ J_1 < 0, J_2 > 0, J' > 0 \]

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite Cu$_3$V$_2$O$_7$(OH)$_2$·2H$_2$O

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Frustrated ferromagnetism

PHYSICAL REVIEW B 82, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite \( \text{Cu}_3\text{V}_2\text{O}_7(\text{OH})_2 \cdot 2\text{H}_2\text{O} \)

O. Janson,\textsuperscript{1,*} J. Richter,\textsuperscript{2} P. Sindzingre,\textsuperscript{3} and H. Rosner\textsuperscript{1,3}

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![Diagram of the magnetic model](image)

DFT gets it right!

\( J_1 < 0, J_2 > 0, J' > 0 \)
Other systems

Semiclassical analysis of a magnetization plateau in a 2D frustrated ferrimagnet

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(Dated: November 9, 2016)

arXiv:1610.03135

Kagome geometry

Spin-current pattern

FIG. 1. Proposed Hamiltonian for volborthite. The blue dots represent spin-1/2 copper ions and the line segments represent Heisenberg couplings. \( J_1 < 0 \) is ferromagnetic while \( J_2 > 0 \) and \( J' > 0 \) are antiferromagnetic. The distances between adjacent unit cells is slightly anisotropic, with \( a = 5.84 \) Å and \( a' = 6.07 \) Å [10]. Capital letters label the sublattices.

FIG. 6. Schematic quantum phase diagram at \( J' = 0.5|J_1| \). The UUD state breaks no symmetries, the gapped chiral liquid (CL) phase only breaks chiral symmetry, and the gapless cone state breaks both chiral and a \( U(1) \) symmetry combining translation and spin rotation. The thick solid lines and dashed lines represent first- and second-order transitions respectively. We did not investigate the nature of the transition between the chiral liquid and cone phases represented by the red line. In a 3D phase diagram like that of Fig. 4 that includes the applied field, the chiral liquid phase would appear as a thin tube around the stabilization curve where the two sheets meet.

FIG. 7. Spin-current configuration in the chiral liquid phase. The magenta arrows indicating the direction of spin current flow; in the orthogonal ground state the flow is reversed. The spin current on the diagonal bonds, represented by thicker arrows, is larger by a factor of \( \sqrt{2} \) and determines the net current flow. The ground state is chiral and spontaneously breaks the lattice symmetry of reflection about the dotted line.
Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

- Spin-1 model with featureless Mott ground state at large $D > 0$ [$S^z_{r} = 0$]
- Triangular lattice: two-fold degenerate spectrum, at $+Q$ and $-Q$

\[
H = \sum_{\langle r,r' \rangle} J (S^x_r S^x_{r'} + S^y_r S^y_{r'} + \Delta S^z_r S^z_{r'}) + D \sum_r (S^z_r)^2
\]

1. Toy problem of two-spin exciton. Derive Schrödinger eqn for the pair wave function $\psi$

\[
|\text{ex}\rangle = \sum_{n \neq m} \psi_n,m |n,m\rangle \text{ where } |n,m\rangle = \frac{1}{2} S^+_n S^-_m \Pi_j |0\rangle_j
\]

Solution which is **odd** under inversion is the first instability when approaching from large-$D$ limit. Indicates chiral Mott phase.

[Single-particle condensation occurs at $D=3J$.]
Is this a generic feature of the Mott-to-Superfluid transition?

Zhentao Wang (U Tenn), Adrian Feiguin (Northeastern U), Cristian Batista (U Tenn), OAS

Spin-1 model with featureless Mott ground state at large $D > 0$
Triangular lattice

$$H = \sum_{\langle r, r' \rangle} J (S^x_r S^x_{r'} + S^y_r S^y_{r'} + \Delta S^z_r S^z_{r'}) + D \sum_r (S^z_r)^2$$

2. Solve Bethe-Salpeter eqn.

Preliminary result $\Delta$
Conclusions

**Mott -> superfluid** transition in frustrated lattice requires $U(1) \times \mathbb{Z}_2$ breaking.

This proceeds via intermediate **spin-current** (**chiral Mott**) phase (breaking $\mathbb{Z}_2$ only).

Spontaneously breaks inversion/time reversal.

All single particle excitations are gapped.

Thank you!
Side remark: **spin-1/2** triangular lattice AFM *is different*

![Graph showing magnetic phases](image)

**Ground states of spin-1/2 triangular antiferromagnets in a magnetic field**

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*R=1-J'/J*

**S=1/2 vs S=1 difference**

for the ordered 2d phase!!!