Triangular lattice antiferromagnets -- open questions

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Outline

• Motivation (Cs$_2$CuBr$_4$) and theoretical progress
  ▶ crucial role of spatial anisotropy
  • Cs$_2$CuCl$_4$ and Cs$_2$CuBr$_4$
  • organic materials (t, t’, U...)

• Phase diagram of spatially anisotropic Heisenberg model
  ▶ Large-S analysis of interacting spin waves near $J’=J$
  ▶ Approach from one dimension, $J’ << J$

• Open questions
Experiment: $M=1/3$ magnetization plateau in $\text{Cs}_2\text{CuBr}_4$

★ Observed in $\text{Cs}_2\text{CuBr}_4$ (Ono 2004, Tsuji 2007) $J'/J = 0.5-0.75$
but not $\text{Cs}_2\text{CuCl}_4$ [$J'/J = 0.34$]

$S=1/2$

9 experimental phases $\nu S$
3 theoretical

Fortune et al, Phys. Rev. Lett. 102, 257201 (2009)
Progress in one dimensional J$_1$-J$_2$ chain (zig-zag ladder)

Okunishi, Tonegawa JPSJ (2003)
Hikihara et al PRB (2010)

M=1/3 plateau

agrees with Oshikawa, Yamanaka, Affleck argument (PRL 2007):

\[ p \, S \, (1 - M) = \text{integer} \]

\( p = \text{period, } S = \text{spin, } M = \text{magnetization:} \)

M=1/3, p=3 possible for all S

plateau is centered around J$_2$ = J$_1$/2 point for S > 1/2; semi-classical spin wave expansion is possible there (OS 2009)
$D = 2$

- surprisingly complex phase diagram of spatially anisotropic triangular lattice antiferromagnet
  - no definite conclusions from numerical studies yet...
- connections with interacting boson system
  - Superfluids
  - Mott insulators
  - Supersolids
    
    Nikuni, Shiba 1995
    Heidarian, Damle 2005
    Wang et al 2009
    Jiang et al 2009
    Heidarian, Sorella, Becca 2009
    Tay, Motrunich 2010
Spatially anisotropic model near $J' = J$

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i S_i^z$$

$J' \neq J$

Umbrella state: favored classically; energy gain $(J-J')^2/J$

Planar states: favored by quantum fluctuations; energy gain $J/S$

The competition is controlled by dimensionless parameter

$$\delta = S (J - J')^2 / J^2$$

- Technical formulation: spatial anisotropy $J-J'$ causes softening of interacting (including $1/S$ correction) spin waves
Sketch of phase diagram ($J-J' \ll J$)

- Fully polarized state
- "Exact" dilute boson calculation
- $V$ incommensurate: fan
- (standard) umbrella
- Supersolid
- Superfluid $S^+_r \sim S e^{iQ \cdot r}$, $S^z_r = \text{const}$
- UUD plateau
- BEC $k = 0$
- BEC $k \neq 0$
- 2 low-energy gapped modes
- Distorted umbrella (2)
- Supersolid
- Planar supersolid
- Commensurate
- Distorted umbrella (1)
- Supersolid
- Commensurate supersolid
- Incommensurate
- Zero-field spiral
- $\delta \sim S(1 - J'/J)^2$

Supersolid: $S^+_r \sim S \cos[Q \cdot r]$, $S^z_r \sim S - S \cos^2[Q \cdot r]$

Questions:
- Phases?
- Transitions?

Alicea, Chubukov, OS PRL 2009
Variational wave function calculation
Tay, Motrunich PRB 2010

Phase diagram for smaller J’/J?
J’ << J: weakly coupled Heisenberg chains in magnetic field

- **Non-frustrated inter-chain coupling**
  \[ \vec{S}_x \cdot \vec{S}_{x+1} \rightarrow N_{x+1}^x N_{x+1}^x + N_{y}^y N_{y}^y + N_{z}^z N_{z}^z \]
  - Most relevant
  - Less relevant
  \[ 2\pi R^2 < 1/(2\pi R^2) \]
  Spins order in the plane perpendicular to the direction of magnetic field (z): umbrella / cone / spin-flop states

- **Frustrated inter-chain coupling**
  \[ \vec{S}_{x,y} \cdot (\vec{S}_{x,y+1} + \vec{S}_{x+1,y+1}) \rightarrow N_{y}^x \partial_x N_{y+1}^x + N_{y}^y \partial_y N_{y+1}^y + \sin(\delta) S_{z}^{\pi-2\delta(y)} S_{\pi+2\delta(y+1)}^{z} \]
  - Less relevant
  - Most relevant (small to intermediate fields)
  \[ 1 + 2\pi R^2 > 1/(2\pi R^2) \]
  ★ Frustration promotes collinear SDW order
$J' \ll J$ limit

"collinear" SDW

"cone"

polarized

$0 \quad uud \quad 0.9 \quad 1 \quad h/h_{sat}$
$J' \ll J$ limit

- "collinear" SDW
- "cone"
- polarized

$0 \leq \frac{J'}{J} \leq 1$

$\frac{h}{h_{\text{sat}}} = 1$

$\frac{3}{5}$

$\frac{1}{3}$

$J'/J = 0$

$J'/J = 1$

Katsura, OS, Balents PRB 2010
J’ << J limit to J’ = J point...

J’/J = 1

J’/J = 0

incomm. planar

comm. planar

UUD plateau

2 low-energy

BEC $k \neq 0$

distorted umbrella (1)

distorted umbrella (2)

h/h_{sat}

0.9

1

3/5

1/3

“collinear” SDW

polarized

“cone”

longit. sdw

planar

BEC $k \neq 0$

2 low-energy
Global phase diagram

*Hypothesis:* 1/3 plateau extends for all 0 < J’/J < 1; other magnetization plateaux terminate above some critical J’/J ratio.

**Question:** how many phases are there? magnetization plateaux?
Experimental relevance

$\frac{J'}{J} = 0$

$\frac{J'}{J} = 1$

$\frac{h}{h_{\text{sat}}}$

"collinear" SDW

"cone"

inter-layer exchange $\frac{J''}{J}$

Cs$_2$CuBr$_4$

plateau - yes

large $\frac{J''}{J}$ favors classical cone order!

Cs$_2$CuCl$_4$

no plateau

UUD plateau

incomm. planar

distorted umbrella (2)

distorted umbrella (1)

comm. planar

longitudinal sdw

CAF
Conclusions

★ Magnetization plateau persists for all J’/J (?)
  - semiclassical interacting spin waves near J - J’ << J
  - 1d scaling + symmetry arguments near J’ << J
★ Many interesting magnetically ordered phases
  - global phase diagram of triangular antiferromagnet ?
  - Longitudinal SDW (?)
    • S=1/2 vs S=1 (?)
  - plateau for ferromagnetic J₁? [LiCuVO₄]
★ Many open experimentally relevant questions, excellent problem for numerical studies