Quasi-one-dimensional spin nematic states and their excitations

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Outline

• Very brief intro
  - emergence of composite orders from competing interactions

• Nematic vs SDW in LiCuVO$_4$
  ✓ spin nematic: “magnon superconductor”
  ✓ collinear SDW: “magnon charge density wave”

• Volborthite kagome antiferromagnet
  – experimental status - magnetization plateau
  – Nematic, SDW and more
  – Field theory of the Lifshitz point

• Conclusions
Emergent nematic (Ising) order parameters

\[ \sigma = \vec{N}_1 \cdot \vec{N}_2 = \pm 1 \]
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LiCuVO$_4$ : magnon superconductor?

estimates:

$J_1 = -1.6$ meV
$J_2 = 3.9$ meV (subject of active debates)
$J_5 = -0.4$ meV
High-field analysis: condensate of bound magnon pairs

\[ \langle S^+ \rangle = 0 \quad \langle S^+ S^+ \rangle \neq 0 \]

Ferromagnetic \( J_1 < 0 \) produces attraction in real space

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Chubukov 1991
Kecke et al 2007
Kuzian and Drechsler 2007
Hikihara et al 2008
Sudan et al 2009
Zhitomirsky and Tsunetsugu 2010
Magnon binding

1-magnon

2-magnon bound state

\[ E-E_{FM} = \varepsilon_1 + h \]
\[ E-E_{FM} = \varepsilon_2 + 2h \]

Formation of molecular fluid
For \( d>1 \) at \( T=0 \) this is a molecular BEC
= true spin nematic
Hidden order

No dipolar order

\[ \langle S_i^+ \rangle = 0 \]
\[ \langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi} \quad S^z = 1 \text{ gap} \]

Nematic order

\[ \langle S_i^+ S_{i+a}^+ \rangle \neq 0 \]

Magnetic quadrupole moment
Symmetry breaking \( U(1) \to Z_2 \)

can think of a fluctuating fan state

nematic director
LiCuVO$_4$ experiment: collinear SDW along $\mathbf{B}$

Hagiwara, Svistov et al, 2011

Buttgen et al 2012, 2014
Evidence of a Bond-Nematic Phase in LiCuVO₄

M. Mourigal,¹,² M. Enderle,¹ B. Fák,³ R. K. Kremer,⁴ J. M. Law,⁴,* A. Schneidewind,⁵ A. Hiess,¹,† and A. Prokofiev⁶,⁷

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

**SF** = spin flip, ΔS = 1
**NSF** = no spin flip, ΔS = 0

FIG. 3 (color online). Polarized cross sections measured at T = 70 mK for the magnetic reflections Q = (1, k₁C, 0) with H∥c [left panels, (a)–(c)] and Q = (0, −k₁C, 1) with H∥a [right panels, (d)–(f)].
1d $J_1$-$J_2$ chain is only *quasi*-spin-nematic

- No true condensation [U(1) breaking] in d=1.
- Inter-chain interaction is crucial for establishing symmetry breaking in d=2.
- Need to study weakly coupled “superconducting” chains

$J_1 < 0$ (ferro)
$J_2 > 0$, $J' > 0$ (afm)
in magnetic field

Sato et al 2013
Starykh and Balents 2014
Inter-chain interaction  
\[ H_{\text{inter-chain}} = \sum_y \int dx \, \vec{S}_y \cdot \vec{S}_{y+1} \sim \sum_y \int dx \, S^+_y S^-_{y+1} + S^z_y S^z_{y+1} \]

Superconducting analogy: single-particle (magnon) tunneling between magnon superconductors is strongly suppressed at low energy (below the single-particle gap)
\[ H_{\text{inter}} \perp = \sum_y \int dx \, J' \langle S^+_y(x) S^-_{y+1}(x+1) \rangle_{\text{nematic ground state}} \rightarrow 0 \]

Superconducting analogy: fluctuations generate two-magnon (Josephson coupling) tunneling between chains. They are generically weak, \( \sim J_1(J'/J_1)^2 \ll J' \), but responsible for a true two-dimensional nematic order
\[ H_{\text{nem}} \sim (J'/J_1)^2 \sum_y \int dx \, \left[ T^+_y(x) T^-_{y+1}(x) + \text{h.c.} \right] \]
\[ T^+_y(x) \sim S^-_y(x) S^-_{y}(x+1) \]

At the same time, density-density inter-chain interaction does not experience any suppression. It drives the system toward a two-dimensional collinear SDW order.
\[ S^z_y = M - 2n_{\text{pair}} = M - \tilde{A}_1 e^{i \frac{\sqrt{2\pi}}{\beta} \varphi^+_y(x)} e^{i k_{\text{sdw}} x} \]
\[ H_{\text{inter-chain}} \perp = H_{\text{sdw}} \sim J' \sum_y S^z_y S^z_{y+1} \sim J' \sum_y \int dx \, \cos \left[ \frac{\sqrt{2\pi}}{\beta} (\varphi^+_y - \varphi^+_{y+1}) \right] \]

Away from the saturation, SDW is more relevant [and stronger, via \( J' \gg (J')^2/J_1 \)] than the nematic interaction: coupled 1d nematic chains order in a 2d SDW state.
Simple scaling

\[ H_{\text{nem}} \sim (J'^2 / J_1) \sum_y \int dx \left[ T^+_y(x) T^-_{y+1}(x) + \text{h.c.} \right] \]

- describes kinetic energy of magnon pairs, linear in magnon pair density \( n_{\text{pair}} \)

\[ H_{\text{inter-chain}} = H_{\text{sdw}} \sim J' \sum_y S^z_y S^z_{y+1} \sim J' \sum_y \int dx \cos \left[ \frac{\sqrt{2\pi}}{\beta} (\varphi^+_y - \varphi^+_{y+1}) \right] \]

- describes potential energy of interaction between magnon pairs on neighboring chains, quadratic in magnon pair density \( n_{\text{pair}} \)

- Competition \( \frac{(J')^2}{J_1} n_{\text{pair}} \sim J' n_{\text{pair}}^2 \), hence \( n_{\text{pair}}^* \sim J' / J_1 \)

- Hence:
  - Spin Nematic near saturation, for \( n_{\text{pair}} < n_{\text{pair}}^* \)
  - SDW for \( n_{\text{pair}} > n_{\text{pair}}^* \)
T=0 schematic phase diagram of weakly coupled nematic spin chains

Cautionary remark: maybe impurity effect

BEC physics

Spin Nematic

1/2 - O(J'/J)

SDW

Fully Polarized

cf: Sato, Hikihara, Momoi 2013

J_1/J_2 = -0.5
J_3/J_2 = J_1/J_2 = 0.005

\( T_{SDW}/J_2 \) and \( T_{SN}/J_2 \)
Intermediate Summary

• Interesting magnetically ordered states: SDW and Spin Nematic
  - Gapped $\Delta S=1$ excitations (no usual spin waves!)
  - 2d Nematic very near the full saturation
  - 2d SDW from nematic chains, occupies most of the phase diagram

toy problem of “magnon high $T_c$”
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Volborthite
Volborthite’s timeline

**Formula:** \( \text{Cu}_3(\text{V}_2\text{O}_7)(\text{OH})_2 \cdot 2\text{H}_2\text{O} \)

**System:** Monoclinic

**Hardness:** 3½

**Name:** Named after Alexander von Volborth (1800–1876), Russian paleontologist, who first noted the mineral.

A secondary mineral found in the oxidized zones of vanadium-bearing hydrothermal deposits.

At least two different monoclinic space-group variants (C2/m, C2/c) seem to be stable at ambient temperature.

Visually similar to vésigniéite.

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**2001**

Quantum spin liquid?!

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**2009**

Impurity ordering at low T? Magnetization steps?

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**2012**

Magnetic order!

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**2014**

Magnetization plateau

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**time = material quality**
2014: huge plateau!

H. Ishikawa...M. Takigawa...Z. Hiroi, unpublished, 2014

High-field magnetization

more different $MH$ curves in a pile of 50 large “thick” arrowhead-shaped crystals

30 days growth

Huge 1/3 plateau!

further optical meas.

@ Takeyama lab

It survives over 120 T!

Kagome plateau or ferrimagnetic state?

coupled to lattice, but already distorted

high-field mag. meas.

@ Tokunaga & Kindo labs
Phase diagram

small plateau’s onset field of 27 Tesla, relative to J ~ 100 K, suggest the presence of ferromagnetic exchange interactions

H. Ishikawa et al, unpublished
Frustrated ferromagnetism

PHYSICAL REVIEW B 82, 104434 (2010)

Coupled frustrated quantum spin-$\frac{1}{2}$ chains with orbital order in volborthite Cu$_3$V$_2$O$_7$(OH)$_2$·2H$_2$O

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DFT gets it right!

\[ J_1 < 0, \ J_2 > 0, \ J' > 0 \]
Ferrimagnetic state

$J_1 < 0, J_2 > 0, J' > 0$

J$_1$ FM, J$_2$ AF

polarized chains?!
Spin chain redux

Frustrated ferromagnetic chain

\[
H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z
\]

H/(|J_1|+J_2)

FM

quasi-spin-nematic
Quasi-1d nematic

1d J1-J2 chain is only quasi-spin-nematic

power-law correlations

\[ \Psi \sim (S^+)^2 : \text{spin-nematic} \]

\[ \phi \sim S^z e^{i q x} : \text{SDW} \]

\[ h/J_2 \]

\[ J_1/J_2 \]

Hikihara et al., 2008

Sudan et al., 2009

FIG. 2. Color online

Typical behaviors of various correlation functions in the vector chiral phase,

2
3
2
3
−
>
s
r
r
J
J
1
2
1
2
SDW
(p=4)

Vector Chiral Order

SDW
(p=3)

quadrupolar

octupolar

1
2
3
4
SDW (p=2)

SDW

PS: SDW

IN

F

T

SDW3

VC

Q

(a)
A QCP parent?

Frustrated ferromagnetic chain

\[ H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z \]

\[
\begin{array}{c}
\text{H}/(|J_1|+J_2) \\
\text{“Lifshitz”}
\end{array}
\]

\[
\begin{array}{c}
\text{QCP} \\
\text{FM}
\end{array}
\]

\[
\begin{array}{c}
\text{quasi-spin-nematic}
\end{array}
\]
Lifshitz Point

- Unusual QCP: order-to-order transition
- Effective action - NLσM for unit vector \( \mathbf{m} \)

\[
S = \int dx d\tau \left\{ i\sigma A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h\hat{m}_z \right\}
\]

\[
A_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}
\]

Berry phase \( \delta \propto |J_1| - 4J_2 \)

All properties near Lifshitz point obey “one parameter universality” dependent upon \( u/K \) ratio.
Lifshitz Point

\[ S = \int dx d\tau \{ isA_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - \hbar \hat{m}_z \} \]

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

\[ x \to \sqrt{\frac{K}{|\delta|}} x \quad \tau \to \frac{K}{\delta^2} \tau \]

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ isA_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \overline{h} \hat{m}_z \} \]

Large parameter: saddle point!

\[ v = \frac{u}{K} \quad \overline{h} = \frac{hK}{\delta^2} \]
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ isA_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \bar{h}\hat{m}_z \} \]

\(-1 < v < -1/4\) derives from quantum fluctuations

Large \(S \gg 1\): \(v \sim -3/(2S) < 0\)

\(S=1/2\): \(v = -5/8\)

\(\bar{h}\)

order parameter discontinuity

\((\sqrt{4|v|} - 1)/|v| \ll 1\) for \(v \approx -1/4\)

\(h_c = \frac{\delta^2}{8K\sqrt{|v|(1 - \sqrt{|v|})}}\)

\(-1 < v < -\frac{1}{4}\)

local instability of FM state

(1-magnon condensation)

\(\hat{m} = \begin{pmatrix} |\Psi|\cos(qx + \phi) \\ \pm |\Psi|\sin(qx + \phi) \end{pmatrix} \sqrt{1 - |\Psi|^2}\)

first order
Metamagnetic endpoint?

\[ \frac{h}{K} \]

\[ \varepsilon_{FM} = \varepsilon_{cone} \]

\[ \epsilon_1 = 0 \]

\[ \varepsilon_{FM} - \varepsilon_{cone} \sim a\delta^2 \]

Quantum corrections penalize \( E_{cone} \) but not \( E_{FM} \)

\[ \Delta \varepsilon_{cone} = +f(v)\delta^{5/2} \]
Metamagnetic endpoint?

\[ S = \int dx d\tau \left\{ i s A_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

\[ \hat{m} = \sqrt{2 - n_1^2 - n_2^2 (n_1 \hat{e}_1(x) + n_2 \hat{e}_2(x)) + (1 - n_1^2 - n_2^2) \hat{e}_3(x)} \]

\[ \hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}^{sp}(x) \]

\[ \eta = n_1 + i n_2 \quad \bar{\eta} = n_1 - i n_2 \]

\[ S = S_{sp} + \int dx d\tau \left\{ \bar{\eta} \partial_\tau \eta + H(\bar{\eta}, \eta) \right\} + O(\eta^3) \]

Bogoliubov transformation gives correction to GS energy
Metamagnetic endpoint?

\[ \frac{h}{K} \]

\[ \mathcal{E}_{FM} = \mathcal{E}_{cone} \]

\[ \epsilon_1 = 0 \]

\[ \mathcal{E}_{FM} - \mathcal{E}_{cone} \sim a\delta^2 - f(v)\delta^{5/2} \]

Corrected first order curve bends slightly downward to intersect second order line
Instabilities

- Choose $E_{FM} = 0$

But the bound states cannot get arbitrarily deep - low density approximation is violated
A natural speculation

Instabilities

- Choose $E_{FM} = 0$

Expect that $n$-boson bound states bend with increasing $n$ to approach continuum line.
Summary

Lifshitz point is a "parent" of many phases

\[ S = \int dx d\tau \left\{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]
Spin chains keep showing up in unexpected places

✓ Nematic physics of frustrated ferromagnets

✓ Explored Lifshitz point as a “parent” for multipolar states and metamagnetism
\[ S = \int dx d^{d-1} y d\tau \left\{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + c |\partial_y \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

- Rescaling:

\[
x \rightarrow \sqrt{\frac{K}{|\delta|}} x \quad \tau \rightarrow \frac{K}{\delta^2} \tau \quad y \rightarrow \frac{\sqrt{cK}}{\delta} y
\]

\[
S = \frac{\sqrt{K^d C^{d-1}}}{\delta^{d-1/2}} \int dx d^{d-1} y d\tau \left\{ i s A_B [\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \bar{h} \hat{m}_z \right\}
\]

\[\therefore \text{ Similar theory applies in } d>1, \text{ and very similar conclusions apply}\]