Phases of spin chains with uniform Dzyaloshinskii-Moriya interactions

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Outline

Spin 1/2 chains with uniform DM interaction

Electron spin resonance

Phase diagram of the quantum model with orthogonal magnetic field

Weakly coupled chains

Conclusions
Dzyaloshinskii-Moriya (DM) interaction

\[ D_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j \]

- Reduces symmetry to U(1) [rotations about D axis]
- Easy plane anisotropy (perp. to D)
- Promotes magnetic order
- Generically stabilizes incommensurate non-collinear (spiral) states
Cs$_2$CuCl$_4$: uniform Dzyaloshinskii-Moriya interaction

\[ \vec{D}_{ij} \cdot \vec{S}_i \times \vec{S}_j \]

Focus on the in-chain DM: for a given chain (y,z) vector D is constant

\[ D_{y,z} = (-1)^y D_c \hat{c} + (-1)^z D_a \hat{a} \]
- Spin chain material $K_2CuSO_4Br_2$.

Spin chain along $a$ axis. 
Intra-chain exchange $J=20.5K$, 
Inter-chain exchange exchange $J’=0.03K$.


FIG. 2. (Color online) Comparison of the Dzyaloshinskii-Moriya vectors in $Cs_2CuCl_4$ and $K_2CuSO_4Br_2$. Perspective view along the spin chains.
ESR - electron spin resonance

- Simple (in principle) and sensitive probe of magnetic anisotropies (and, also, $\mathbf{q}=0$ probe: $S = \Sigma_r S_r$)

$$I(h, \omega) = \frac{\omega}{4L} \int dt e^{i\omega t} \langle [S^+(t), S^-] \rangle$$

- For SU(2) invariant chain in paramagnetic phase
  $$I(H, \omega) \sim \delta(\omega-H) m(H) \text{ [Kohn’s Th]}$$

Oshikawa, Affleck PRB 2002

Zeeman $\mathbf{H}$ along $z$, microwave radiation $\mathbf{h}$ polarized perpendicular to it.
Resonance line is significantly modified with lowering the temperature; modification is strongly anisotropic with respect to field. The lowest temperature $T=1.3\, K$ is still twice higher than ordering $T_N$. 

ESR data (Povarov and Smirnov, Kapitza Institute)
Cs$_2$CuCl$_4$  ESR data: T=1.3 K  Poverov et al, 2011

- **H along b-axis**
  - gap-like behavior for $\nu > 17$ GHz
  - $2\pi\hbar\nu = \sqrt{(g_b\mu_B H)^2 + \Delta^2}$
  - loss of intensity for $\nu < 17$ GHz

- **H along a-axis:** splitting of the ESR line

\[ g_b = 2.08 \]
\[ g_a = 2.20 \]
Continuum in magnetic field

\[ H \neq 0 \]

\[ S_{\perp} \] transverse structure factor

\[ \Delta S = 1 \]

Dender et al, PRL 1997

**Explanation I: H along DM axis**

\[ H = \sum_{x,y,z} JS_{x,y,z} \cdot S_{x+1,y,z} - D_{x,y,z} \cdot S_{x,y,z} \times S_{x+1,y,z} - g\mu_B H \cdot S_{x,y,z} \]

- **Chain**
- **Uniform DM along the chain**
- **Magnetic field**

Unitary rotation about z-axis:

\[ S^+(x) \rightarrow S^+(x) e^{i(D/J)x}, S^z(x) \rightarrow S^z(x) \]

- Removes DM term from the Hamiltonian (to D^2 accuracy)
- **Boosts** momentum to D/(J a0)

\[ q = 0 \rightarrow q = D/(Ja_0) \Rightarrow 2\pi\hbar R/L = g\mu_B H \pm \pi D/2 \]

Dotted lines: D=0 picture

Oshikawa, Affleck 2002

Chiral probe: **ESR probes right-** and **left-** moving modes (spinons) independently
Spectrum shift due to the **uniform** DM

\[ \mathcal{H} = \sum_n J(S_n \cdot S_{n+1}) + (D \cdot [S_n \times S_{n+1}]) - \mu_B g(H \cdot S_n) \]

Gangadharaiyah, Sun, Starykh, PRB 78 054436 (2008)

Spin transformation \( S_n^+ = \tilde{S}_n^+ e^{i\alpha n} \quad \alpha = -D/J \) excludes DM interaction, but **results in the spectrum shift** by momentum \( D/J \). This leads to **two** ESR peaks.

\[ D \parallel H \parallel z \]

Karimi, Affleck, PRB 2011
Explanation II: arbitrary orientation
Field theory in terms of spin currents $\mathbf{M}$

$$H = \frac{2\pi v}{3} [(\bar{M}_R^2 + (\bar{M}_L^2)] - \frac{vD}{J} [M_R^d - M_L^d] - g\mu_B H [M_R^z + M_L^z]$$

unperturbed chain uniform DM along the chain magnetic field

Uniform DM produces internal momentum-dependent magnetic field along $\mathbf{d}$-axis

- Total field acting on right/left movers $g\mu_B \bar{H} \pm \hbar \nu \bar{D} / J$
- Hence ESR signals at $2\pi \hbar \nu_{R/L} = |g\mu_B \bar{H} \pm \hbar \nu \bar{D} / J|$  
- Polarization: for $H=0$ maximal absorption when microwave field $h_{mw}$ is perpendicular to the internal (DM) one. Hence $h_{mw} \parallel \mathbf{b}$ is most effective.

Povarov et al, PRL 2011
Gangadharaih, Sun, OS, PRB 2008
Explanation III: arbitrary orientation

- General orientation of $\mathbf{H}$ and $\mathbf{D}$
- 4 sites/chains in unit cell

\[ (2\pi\hbar\nu_R)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a + (-1)^y \pi D_a/2]^2 + [g_c\mu_B H_c + (-1)^y \pi D_c/2]^2, \]

\[ (2\pi\hbar\nu_L)^2 = (g_b\mu_B H_b)^2 + [g_a\mu_B H_a - (-1)^y \pi D_a/2]^2 + [g_c\mu_B H_c - (-1)^y \pi D_c/2]^2. \]

\[ D_a/(4\hbar) = 8 \text{ GHz} \]
\[ D_c/(4\hbar) = 11 \text{ GHz} \]

0.3 Tesla
0.4 Tesla
$D \sim J/10$

- for $\mathbf{H}$ along b-axis only: the “gap” is determined by the DM interaction strength

\[ \Delta = \frac{\pi}{2}\sqrt{D_a^2 + D_c^2} \rightarrow (2\pi\hbar) 13.6 \text{ GHz} \]

T-dependent line width

This explanation suggests:
1) ESR absorption in the absence of H $\nu \sim \sqrt{D_a^2 + D_c^2}$
2) strong polarization dependence in zero field

The largest absorption occurs when microwave field $h(t)$ is lined along crystal b-axis, $h \parallel b$ [so that it is perpendicular to the $\mathbf{D}$ vector in $a$-$c$ plane]
Extension to 2d spin liquids

Spinon magnetic resonance of quantum spin liquids

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We describe electron spin resonance in a quantum spin liquid with significant spin-orbit coupling. We find that the resonance directly probes spinon continuum which makes it an efficient and informative probe of exotic excitations of the spin liquid. Specifically, we consider spinon resonance of three different spinon mean-field Hamiltonians, obtained with the help of projective symmetry group analysis, which model a putative quantum spin liquid state of the triangular rare-earth antiferromagnet YbMgGaO₄. The band of absorption is found to be very broad and exhibit strong Hove singularities of single spinon spectrum as well as pronounced polarization dependence.

FIG. 1. (Color online) Plot of $2(I(\omega)}/|h|^2$ vs $\omega/\hbar$ for different polarizations $\theta = 0$ (blue dots), $\pi/4$ (orange squares), and $\pi/2$ (green rhombi) for U1A11 state. The insert shows spinon band structure along the high-symmetry path $\Gamma-K-M-K-\Gamma$ in the Brillouin zone. Vertical red line illustrates optical transitions between spinon bands.
Spin 1/2 chains with uniform DM interaction

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Conclusions
Model Hamiltonian

\[ H = J \sum_i \left[ S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right] - \sum_i D \hat{\mathbf{z}} \cdot (\mathbf{S}_i \times \mathbf{S}_{i+1}) - h \sum_i S_i^x, \]

- J > 0, the spins antiferromagnetically interact with nearest neighbor.
- The DM interaction is uniform along chain.
- The external magnetic field is perpendicular to the DM vector.
- Small anisotropy \( \Delta \sim 1 \).
Classical version: chiral soliton lattice

- In classical version, the spin can be treated as vectors with fixed length.

\[ \mathbf{S}_i = S(\sin \theta_i \cos \phi_i, \sin \theta_i \sin \phi_i, \cos \theta_i) \]

\[ H = J \sum_i \left[ S^x_i S^x_{i+1} + S^y_i S^y_{i+1} + \Delta S^z_i S^z_{i+1} \right] - \sum_i D \hat{z} \cdot (\mathbf{S}_i \times \mathbf{S}_{i+1}) - h \sum_i S^x_i, \]

Quantum version

- In quantum mechanics, spins are operators satisfy commutation relation, \([S^\alpha, S^\beta] = i\epsilon_{\alpha\beta\gamma} S^\gamma\).

- Spin-1/2 operator can be mapped to fermionic one by Jordan-Wigner transformation:
  \[ S^+_j = \psi^+_j \exp(i\pi \sum_{k<j} \psi^+_k \psi_k), \]
  \[ S^-_j = \exp(-i\pi \sum_{k<j} \psi^+_k \psi_k) \psi_j, \]
  \[ S^z_j = \psi^+_j \psi_j - 1/2, \]

- Spin Hamiltonian transforms to interaction fermions:
  \[
  H = J \sum_{\langle i,j \rangle} \left[ \frac{1}{2} (S^+_i S^-_j + S^-_i S^+_j) + \Delta S^z_i S^z_j \right].
  \]
  \[
  H_F = J \sum_i \left[ \frac{1}{2} (\psi^+_i \psi_{i+1} + \psi_i \psi^+_i) + \Delta (\psi^+_i \psi_i - \frac{1}{2}) (\psi^+_i \psi_{i+1} - \frac{1}{2}) \right].
  \]
Bosonization

- A low-energy effective field theory, and non-perturbative approach.
  - Jordan-Wigner transformation, $S_j^z = \psi_j^+ \psi_j - 1/2$.
  - Spin operator coincides with density operator for fermions.
  - Fermions with linearized spectrum near two Fermi points,
    $$\psi(x) = R(x)e^{ipFx} + L(x)e^{-ipFx}$$
  - Low-energy excitations:
    described by Fourier components of the density operator
    $$\rho_q = \sum_p \psi_p^+ \psi_{p+q} \quad \text{with } q \text{ close to } 0 \text{ and } \pm 2p_F \quad (2p_F = \pi).$$

- Spin operator in uniform spin current $J_{R/L}$ and staggered magnetization $N$,
  $$\mathbf{S}(x) \rightarrow J_L(x) + J_R(x) + (-1)^{x/a} N(x), \quad a \text{ is lattice spacing}$$
  with $J_L(x) = L^+(x)L(x), \quad J_R(x) = R^+(x)R(x), \quad N(x) = R^+(x)L(x) + L^+(x)R(x)$.

Here $v$ is Fermi velocity.
Hamiltonian in terms of bosonization

- **Low-energy theory** \( S(x) \rightarrow J_L(x) + J_R(x) + (-1)^{x/a} N(x) \),

\[
H = H_0 + V + H_{bs}, \quad H = J \sum_i \left[ S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right] - \sum_i D \hat{z} \cdot (S_i \times S_{i+1}) - h \sum_i S_i^z,
\]

- Isotropic Heisenberg chain

\[
H_0 = \frac{2\pi v}{3} \int dx (J_R \cdot J_R + J_L \cdot J_L),
\]

- \( V \) contains DM interaction and Zeeman term

\[
V = -D \int dx \left( J_R^z - J_L^z \right) - h \int dx \left( J_R^x + J_L^x \right),
\]

- Back-scattering interaction between right- and left- spin current.

\[
H_{bs} = -g_{bs} \int dx \left[ J_R^x J_L^x + J_R^y J_L^y + (1 + \lambda) J_R^z J_L^z \right],
\]

with coupling constant \( g_{bs} \approx 0.23 \times (2\pi v), \) Garate and Affleck (2010)

Exchange and DM-induced anisotropy:

\[
\lambda = c (1 - \Delta + \frac{D^2}{2J^2}).
\]
Chiral rotation

- Perform a chiral rotation $\mathbf{J}_{R/L} = \mathcal{R}(\theta_{R/L})\mathbf{M}_{R/L}$,

\[
\mathcal{R}(\theta_{R/L}) = \begin{pmatrix}
\cos \theta_{R/L} & 0 & \sin \theta_{R/L} \\
0 & 1 & 0 \\
-\sin \theta_{R/L} & 0 & \cos \theta_{R/L}
\end{pmatrix}, \quad \theta_R = \theta_0 + \pi/2, \quad \theta_L = -\theta_0 + \pi/2, \quad \theta_0 \equiv \arctan\left(\frac{-D}{h}\right).
\]

- Simplified Hamiltonian,

\[
H = H_0 + V + H_{bs}, \rightarrow H = \tilde{H}_0 + \tilde{H}_{bs},
\]

\[
\tilde{H}_{bs} = H_A + H_B + H_C + H_\sigma,
\]

\[
H_A = \pi v y_A \int dx (M_R^z M_L^+ e^{it\phi x} - M_R^+ M_L^z e^{-it\phi x} + \text{h.c.}),
\]

\[
H_B = \pi v y_B \int dx (M_R^+ M_L^z e^{-i2t\phi x} + \text{h.c.}),
\]

\[
H_C = \pi v y_C \int dx (M_R^+ M_L^+ + \text{h.c.}),
\]

\[
H_\sigma = -2\pi v y_\sigma \int dx M_R^z M_L^z.
\]

Oscillating factors

\[
t_\phi \equiv \frac{\sqrt{D^2 + h^2}}{v}
\]
Renormalization group (RG) analysis

- RG equations are well-known Kosterlitz-Thouless equations,

\[
\frac{dy_C}{d\ell} = y_C y_\sigma, \quad \frac{dy_\sigma}{d\ell} = y_C^2.
\]

\[
H_C = \pi v y_C \int \ dx (M_R^+ M_L^+ + \text{h.c.}),
\]

\[
H_\sigma = -2\pi v y_\sigma \int \ dx M_R^- M_L^-.
\]

- Ground state is determined by the initial values.

\[
y_C(0) \propto [(1 + \frac{\lambda}{2}) \cos \theta^- - 1 + \frac{\lambda}{2}],
\]

\[
y_\sigma(0) \propto [(1 + \frac{\lambda}{2}) \cos \theta^- - \frac{\lambda}{2}],
\]

\[
C = y_\sigma^2(0) - y_C^2(0).
\]

- The initial values are determined by ratio \(D/h\), and also influenced by anisotropy.

\[
\cos \theta^- = (h^2 - D^2)/(h^2 + D^2)
\]

\[
\lambda = c(1 - \Delta + \frac{D^2}{2J^2}).
\]
\[ \Delta - h/D \] phase diagram

- There are three distinct phases.
  \[ \langle S(x) \rangle = M \hat{x} + (-1)^x \langle N(x) \rangle. \]
  - \text{"N}^z\text{"} \quad \langle N(x) \rangle \propto (-1)^{k_1+1}z. 
  - \text{"N}^y\text{"} \quad \langle N(x) \rangle \propto \cos \theta_0 (-1)^{k_2}y, 
  - Luttinger-liquid (LL): \langle N(x) \rangle = 0,

- Phase boundaries.
  - \text{"N}^z\text{"}-\text{"N}^y\text{"}: \Delta_c^1 = 1 + \frac{1}{2} \left( \frac{D}{J} \right)^2 - \frac{2}{c} \left( \frac{D}{h} \right)^2.
  - \text{"N}^z\text{"}-LL: \Delta_c^2 = 1 + \frac{1}{2} \left( \frac{D}{J} \right)^2 - \frac{2}{c} \left( \frac{1}{1 + 2(D/h)^2} \right).
  - \text{"N}^z\text{"}-LL: \Delta_c^3 = \sqrt{2}.

- Compared to classical phase diagram, LL replaces soliton lattice.

Quantum phase diagram

Classical phase diagram

Garate and Affleck (2010)
Δ − h/D phase diagram

- Large DM interaction promotes the “N^z” state.
- Phase boundary between LL and “N^y” is independent of DM interaction.
- Phase diagrams are verified by numerical study (DMRG and iTEBD).

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The model Hamiltonian is given by:

\[ H = \sum_{x,y} [JS_{x,y} \cdot S_{x+1,y} + J'S_{x,y} \cdot S_{x,y+1}] + D \cdot \sum_{x,y} (-1)^y S_{x,y} \times S_{x+1,y} - h \cdot \sum_{x,y} S_{x,y}, \]

- \( J > 0 \). Also \( J' > 0 \), the interchain exchange interaction, and \( J' \ll J \).
- The DM interaction is uniform along chain, but antiparallel in neighboring chains.
- Real magnetic insulator \( K_2CuSO_4(Br/Cl)_2 \).
- There is no exchange anisotropy, \( \Delta = 1 \).
- Here we only consider magnetic field perpendicular to DM interaction.

Intra-chain exchange \( J=20.5K \), Inter-chain exchange \( J'=0.03K \), DM interaction \( D=0.28K \).

Hamiltonian in terms of bosonization

- **Low-energy theory**  \( S_{x,y} \to J_{yL}(x) + J_{yR}(x) + (-1)^{x/a}N_y(x) \),

\[
\mathcal{H} = \sum_y [\mathcal{H}_0 + \mathcal{V} + \mathcal{H}_{\text{bs}} + \mathcal{H}_{\text{inter}}], \quad \mathcal{H} = \sum_{x,y} [JS_{x,y} \cdot S_{x+1,y} + J'S_{x,y} \cdot S_{x,y+1}] + D \cdot \sum_{x,y} (-1)^y S_{x,y} \times S_{x+1,y} - h \cdot \sum_{x,y} S_{x,y},
\]

\[
\mathcal{H}_0 = \frac{2\pi v}{3} \int dx (J_{yR} \cdot J_{yR} + J_{yL} \cdot J_{yL}),
\]

\[
\mathcal{V} = -h_z \int dx (J_{yR}^z + J_{yL}^z) - h_x \int dx (J_{yR}^x + J_{yL}^x) + (-1)^y D \int dx (J_{yR}^z - J_{yL}^z),
\]

\[
\mathcal{H}_{\text{bs}} = -g_{\text{bs}} \int dx [J_{yR}^x J_{yL}^x + J_{yR}^y J_{yL}^y + (1 + \lambda) J_{yR}^z J_{yL}^z],
\]

\[
\mathcal{H}_{\text{inter}} = J' \int dx N_y \cdot N_{y+1},
\]

\[
S_{x,y} \cdot S_{x,y+1} \to N_y(x) \cdot N_{y+1}(x)
\]

- **Isotropic Heisenberg chain.**

- Contains DM interaction and Zeeman term.

- Back-scattering interaction between right- and left- spin current.

- Most relevant part of interchain coupling.

- **DM-induced anisotropy**  \( \lambda = c' \frac{D^2}{J^2} \), where \( c' = \left( \frac{2\sqrt{2}v}{g_{\text{bs}}} \right)^2 \approx 3.83 \). Garate and Affleck (2010)

- Ground state is determined by \( \mathcal{H}_{\text{inter}} \), which is strongly affected by \( \mathcal{H}_{\text{bs}} \).
Hamiltonian in terms of bosonization

- Perform a chiral rotation

\[ J_{R/L} = \mathcal{R}(\theta_{R/L}) M_{R/L}, \]

\[ \mathcal{R}(\theta_{R/L}) = \begin{pmatrix} \cos \theta_{R/L} & 0 & \sin \theta_{R/L} \\ 0 & 1 & 0 \\ -\sin \theta_{R/L} & 0 & \cos \theta_{R/L} \end{pmatrix}, \]

\[ \theta_R = \frac{\pi}{2} + \theta_0^y, \]

\[ \theta_L = \frac{\pi}{2} - \theta_0^y, \]

\[ \theta_0^y \equiv (-1)^y \tan^{-1} \left[ \frac{D}{\hbar} \right]. \]

- Simplified Hamiltonian,

\[ \mathcal{H} = \sum_y \left[ \mathcal{H}_0 + \mathcal{V} + \mathcal{H}_{bs} + \mathcal{H}_{inter} \right] \rightarrow \mathcal{H} = \sum_y \left[ \tilde{\mathcal{H}}_0 + \tilde{\mathcal{H}}_{bs} + \tilde{\mathcal{H}}_{inter} \right] \]

\[ \tilde{\mathcal{H}}_{bs} = H_A + H_B + H_C + H_\sigma, \]

\[ H_A = \pi v y_A \int dx (M_R^z M_L^+ e^{it_\varphi x} - M_R^+ M_L^z e^{-it_\varphi x} + \text{h.c.}), \]

\[ H_B = \pi v y_B \int dx (M_R^+ M_L^- e^{-i2t_\varphi x} + \text{h.c.}), \]

\[ H_C = \pi v y_C \int dx (M_R^+ M_L^+ + \text{h.c.}), \]

\[ H_\sigma = -2\pi v y_\sigma \int dx M_R^z M_L^z. \quad t_\varphi \equiv \frac{\sqrt{D^2 + \hbar^2}}{v} \]

\[ \tilde{\mathcal{H}}_{inter,\varphi} \propto g_{\varphi_1} \int dx \cos[\sqrt{2\pi}(\varphi_y - \varphi_{y+1})]. \]

\[ \tilde{H}_x = 2\pi v g_x \int dx \mathcal{N}_y^x \mathcal{N}_{y+1}^x, \]

\[ \tilde{H}_y = 2\pi v g_y \int dx \mathcal{N}_y^y \mathcal{N}_{y+1}^y, \]

\[ \tilde{\mathcal{H}}_{inter,\varphi} \propto g_{\varphi_1} \int dx \cos[\sqrt{2\pi}(\varphi_y - \varphi_{y+1})]. \]
\[ h \perp D \]

**RG equations**

- Backscattering RG equations,
  \[ \frac{d y_C}{d l} = y_C y_\sigma, \quad \frac{d y_\sigma}{d l} = y_C^2, \]

- Interchain interaction RG equations,
  \[ \frac{d g_x}{d l} = g_x \left( 1 + y_C + \frac{1}{2} y_\sigma \right), \]
  \[ \frac{d g_y}{d l} = g_y \left( 1 - y_C + \frac{1}{2} y_\sigma \right), \]
  \[ \frac{d g_{\varphi_1}}{d l} = g_{\varphi_1} \left( 1 - \frac{1}{2} y_\sigma \right). \]

- Ground state is determined by the fastest growing \( g \),
  \[ y_C(0) \propto [(1 + \frac{\lambda}{2}) \cos \theta^- - 1 + \frac{\lambda}{2}], \]
  \[ y_\sigma(0) \propto [(1 + \frac{\lambda}{2}) \cos \theta^- - \frac{\lambda}{2}], \]
  \[ C = y_\sigma^2(0) - y_C^2(0). \]

\[ \theta^- \text{ is function of } h \text{ and } D. \]
There are three distinct phases.

- **SDW(z):** \( \langle S_{x,y} \rangle \sim M x + (-1)^{x+y} \Psi_{sdw(z)} z, \)

- **SDW(y):** \( \langle S_{x,y} \rangle \sim M x + (-1)^{x+y} \frac{h}{\sqrt{h^2 + D^2}} \Psi_{sdw(y)} y. \)

- **distorted-cone:**

\[
\langle S_{x,y} \rangle \sim M x + (-1)^{x+y} \Psi_{\text{dist-cone}} \left( \sin[\sqrt{2\pi\hat{\phi}} + t\varphi]x \right.
+ \left. \frac{(-1)^y D}{\sqrt{h^2 + D^2}} \cos[\sqrt{2\pi\hat{\phi}} + t\varphi]y \right).
\]

- **Distinguish strong coupling by the initial values of** \( yC \) **and** \( y\sigma. \)
\[ h \perp D \quad D - h \text{ phase diagram} \]

- Phase boundary between two SDWs is independent of D, when D/J is small.
- Phase boundary between SDW(y) and distorted-cone is linear with \( h/D \sim 1.5 \).
- Distorted-cone is unlikely to realize in real materials with small D/J.

Typical RG flows for SDW(z).

<table>
<thead>
<tr>
<th>Interaction term</th>
<th>Coupling operator ( N_y N_{y+1} )</th>
<th>Coupling constant ( g )</th>
<th>Induced state</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H}_x )</td>
<td>( \lambda )</td>
<td>( g_x )</td>
<td>SDW(z)</td>
</tr>
<tr>
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</tr>
<tr>
<td>( \mathcal{H}_{\text{inter,}\varphi} )</td>
<td>( \lambda )</td>
<td>( g_{\varphi_1} )</td>
<td>Distorted-cone</td>
</tr>
</tbody>
</table>
DM-induced frustration

- Identical-between-chains DM interactions stabilize 2D helical structure.
- Staggered-between-chains DM interactions effectively cancel the transverse interchain coupling.

\[ h \parallel D \parallel z \]

Intra-chain exchange \( J = 20.5 \text{K} \),
Inter-chain exchange \( J' = 0.03 \text{K} \),
DM interaction \( D = 0.28 \text{K} \).

Model: \( h \parallel D \parallel z \)

- Weakly coupled \( S = 1/2 \) AFM Heisenberg chains with a uniform DMI and magnetic field, \( J' \ll J \).

\[
H = \sum_{x,y} [J S_{x,y} \cdot S_{x+1,y} + J' S_{x,y} \cdot S_{x,y+1}] - D \cdot \sum_{x,y} (-1)^y S_{x,y} \times S_{x+1,y} - h \cdot \sum_{x,y} S_{x,y},
\]

\( J, J' \): Intra-chain and inter-chain exchange, and \((-1)^y D\): staggered between chains.

- Low-energy theory:
  
  ➢ Spin operator in spin current \( J_{R/L} \) and staggered magnetization \( N, x - 1 \quad x \quad x + 1 \)

  \( S(x) \to J_L(x) + J_R(x) + (-1)^{x/a} N(x), \quad a \) is lattice spacing

  ➢ DMI \( (\sim (-1)^y D \partial_x \theta) \) can be absorbed into \( H_0 \) by a shift of bosonic field,

  \[
  H = H_0 + H_{\text{inter}}, \quad H_{\text{inter}} = J' \int dx N_y \cdot N_{y+1} \to H_{\text{cone}} + H_{\text{sdw}},
  \]

  \[
  H_{\text{cone}} = J' \int dx (N_y^+ \cdot N_{y+1}^+ e^{i(1)^y D x / v} + \text{h.c.}), \quad H_{\text{sdw}} = J' \int dx N_y^z \cdot N_{y+1}^z
  \]

  Cone: spiral in XY plane \quad \text{Rapidly oscillates when } D \text{ is large.} \quad \text{Collinear SDW}
Strong DMI: quantum fluctuations generate interaction between next-nearest (NN) chains

- Effective interaction between NN chain, no frustration due to DMI,

\[ H_{c,nn} = J_2 \int dx (N_y^+ \cdot N_{y+2}^- + h.c.), \]

with \[ J_2 \sim -f(\Delta) \frac{J'^2}{D}. \]

\( \Delta \): scaling dimension of \( N^+ \), modulated by magnetic field

- Induced state: Cone2N

Magnetic field enhances Cone2N.
Strong DMI: competition between SDW and Cone2N

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Coupling</th>
<th>Coupling</th>
<th>Ordering</th>
<th>Induced</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{sdw}$</td>
<td>$N^z_0 \cdot N^z_{L+1}$</td>
<td>$g_z$</td>
<td>$T_{sdw}$</td>
<td>SDW</td>
</tr>
<tr>
<td>$H_{cun}$</td>
<td>$N^+_y \cdot N^-_y$</td>
<td>$G_z$</td>
<td>$T_{cone2N}$</td>
<td>Cone2N</td>
</tr>
</tbody>
</table>

Ordering temperatures: for $D/J' = 10,$

- Small field, $h/D = 0.005.$
- Large field, $h/D = 5.$

SDW - cone transition

$T_c (mK)$ vs. $M$

$h_c \sim J'$

RG flows: for $D/J' = 10,$
Summary $h \parallel D$

- Strong DMI promotes collinear SDW.
- Magnetic field enhances Cone2N, quantum fluctuation generate $J_2$ among NN chains.
- C-IC phase transition between SDW and Cone2N.

**Strong DMI**

$K_2CuSO_4Br_2$

- 1st chain
- NN chain

**Weak DMI**

$K_2CuSO_4Cl_2$

- 1st chain
Conclusions

Simple model but many interesting properties

Interesting dependence on the angle between h and D

Strong in-chain DM frustrates interchain exchange

Experimental checks are available